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# The Mathematics Teacher

MAY 1954

## *Which way mathematics?*

*A discussion of some pressing problems in mathematics education.*

*Issues in elementary and secondary school mathematics*

HARL R. DOUGLASS

*Mathematics for our time*

R. S. BURINGTON

*Elementary and secondary school training in mathematics*

S. S. CAIRNS

*Which way precollege mathematics?*

KENNETH O. MAY

*The official journal of*

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# *The Mathematics Teacher* is a journal of The National Council of Teachers of Mathematics devoted to the interests of mathematics teachers in the Junior High Schools, Senior High Schools, Junior Colleges and Teacher Education Colleges

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*Which way mathematics?*

## *Issues in elementary and secondary school mathematics<sup>1</sup>*

HARL R. DOUGLASS, *Director of the College of Education,  
University of Colorado, states some issues and problems  
in mathematics teaching as seen by the general educator.*

THE CURRICULUM and our knowledge of how pupils learn cannot remain static. This is true for several reasons.

In the first place, people's needs change materially from time to time. In the mid-twentieth century people need more mathematics and different mathematics than they needed in the mid-nineteenth century. The American family, for one thing, has become a buying unit rather than a producing unit. Besides, practically all Americans participate in greatly increased financial activities. These include income taxes, social security funds, insurance, and investments.

Another factor contributing to the need for evolution in the mathematics curriculum is the current change in personnel in the mathematics class. In 1900 the pupils who completed elementary school and entered high school constituted a select group. Those who graduated from high school were approximately one in twenty. During the subsequent half century the situation has changed. Now nearly 80 per cent of our young people reach the eighth grade. And more than 50 per cent of the nation's youth finish high

school. Those who stayed in school at the turn of the century differed considerably from those who dropped out. Moreover, the differences appeared in such important matters as ability, home background, morale, common education of their parents, the quality of their education in different grades, vocational choice, intellectual interest, and probable future needs. The implication certainly is clear for courses of study and methods of teaching. In today's elementary school we must think about instruction for all youngsters. In today's high school we must plan for the great majority of boys and girls in grades nine and ten and for instruction for two thirds to three fourths of the boys and girls in grades eleven and twelve.

Thirdly, in the elementary schools there has developed in recent years a very great increase in the promotion rate. Formerly nearly 50 per cent of the pupils were retarded one year, and a considerable number two years, before they reached the seventh grade. Today it is common practice to promote all youngsters at the end of each year's stay in the elementary school, regardless of their mastery of arithmetic for the grade in which they have been. This brings about in some grades a range of four or five or even six years in pupils' mastery of elementary mathematics. In the fifth grade, for ex-

<sup>1</sup> The author is indebted for suggestions to the following colleagues at the University of Colorado: Harold Anderson, Assistant Professor of Education; Burton W. Jones, Professor of Mathematics; Marie Mehl, Assistant Professor of Education.

ample, there may be youngsters at the fourth-grade level of achievement in arithmetic. In the same room there may be other pupils who could easily do sixth-grade or seventh-grade mathematics. The need for adapting the work to fit individual pupils has accordingly become much greater.

A fourth change involves new theories and new knowledge about the nature of learning. We know now, for example, that the transfer of understandings, skills, and other outcomes of learning from an academic field to uses in life depends very largely upon the degree to which the material was learned in relation to the applications in life. We know also that this principle applies more to pupils of inferior or mediocre intelligence than to youngsters of superior intelligence. Besides, we know that the understanding of mathematics depends, at least in part, and in great part for most youngsters, upon orientation concerning its usage. Furthermore, we know that children's efforts to learn mathematics depend on the interest they have in the subject. In other words, effort is exerted in proportion to interest, and interest is proportional to the use and value of the subject, as the child sees it and enjoys it.

These developments have brought about new problems. And they have accentuated some old problems. Indeed, in recent years many of our older problems have become ever more important, and some of them are now quite critical. In the following paragraphs eleven of the more urgent issues are presented and briefly discussed.

*1. How rapidly should young people be pushed in the study of arithmetic?*

The results of over-acceleration in arithmetic should be obvious. Many children have had unhappy experiences with arithmetic. They feel inadequate in quantitative situations; they fear figures; they dread computation. Eventually they develop a bad attitude toward mathematics in general. This has been noticed by many teachers in the upper grades and in col-

lege. This attitude is not easily changed. Because of their earlier experiences with arithmetic and algebra, a considerable number of young people choose not to continue the study of mathematics.

On the other hand, there are some schools in which progress in arithmetic is not overemphasized. By the time the pupils in these schools reach the eighth grade, they seem to have mastered arithmetic better than the young people in those schools in which formal instruction in algorithms mechanically taught was begun earlier.

One of the reasons for pushing children beyond their capacity to understand arithmetic has been the demand of parents that their children show "progress" early in their school life. Parents too often insist that pupils be able to add, subtract, multiply, and divide, and be able to recite various tables of number facts, as their parents and grandparents did.

But early memorization is not the answer. And there is a trend toward slowing down a bit in the primary grades and giving more attention to the problem of seeing that pupils do not get lost in the very beginning.

*2. Should mathematics be taught primarily as a set of rules to be learned and drilled, or should the understanding of procedure always be taught?*

There has been a clear-cut trend toward the teaching of meaning. Textbooks, especially those in arithmetic, provide better for the teaching of meaning. However, the degree to which meaning can be grasped depends on the intelligence of the learner; with the duller children concentration upon the mastery of meaning is likely to require an amount of time disproportionate to the results obtainable.

*3. Should mathematics be taught as a separate subject, or should it be combined with other subjects and integrated in a core or a similar type of curriculum?*

Should mathematics be compartmentalized and taught separately from other subjects? To what degree may mathe-

matics be correlated with other subjects around life problems?

It seems to be difficult for teachers to teach all or even a major part of mathematics in connection with other subjects in the curriculum. The life-problem approach may be utilized to a limited degree, but at least after the first two or three grades there is a distinct necessity for a daily class in mathematics alone. This has been the experience of the great majority of teachers of mathematics. It is also believed, however, that life problems and material from other fields, such as business and home economics, for example, should be employed in mathematics classes. A comparison of recent textbooks with those of twenty to thirty years ago will reveal that there has been a strong trend in that direction.

4. *How best can mathematics be adapted to the interest, the ability, and the previous mastery of mathematics of each individual pupil?*

Should pupils be put into sections according to their abilities, capacities, and achievement in arithmetic, or should they be taught in heterogeneous classes? Should there be remedial sections?

This issue is still a very live one. In some schools pupils in the first three grades are placed in an ungraded primary room in which youngsters are grouped according to their capacities and abilities. There each group proceeds at its own rate, regardless of whether its members accomplish during the school year the work for one year, less than one year, or more than one year. There is a fairly widespread, though not at all universal, feeling that adaptation to individual pupils may be better made within heterogeneous groups and without sectioning according to ability and/or achievement. This belief, however, has lost a little ground during the past few years. The spread of the practice of promoting all the pupils every year accentuates the problem. The demands placed upon the teacher to make adaptation to the wide range of capacity, ability, and

status of her pupils have become excessive. At any rate there seems to be a necessity for grouping some of the most backward pupils. Either in separate rooms or in separate groups in a room, these pupils seem to need special and individual instruction.

There also seems to be the necessity in junior high schools for setting up remedial sections for 10, 15, or as high as 20 per cent of those who are retarded in arithmetic. These sections should be under the tutelage of a teacher who is especially skilled in giving remedial instruction. In some such remedial sections the work will need to go back as far as third-grade and fourth-grade arithmetic.

5. *When should algebra and geometry be introduced?*

Opinions on this subject vary.

During the last twenty or twenty-five years in grades five, six, seven, and eight there has been some increase in the amount of algebra and geometry taught. The content at this level is primarily spatial concepts, plane and solid figures, and formulas. Computations appropriate to the figures predominate; there is little, if any, reference to formal proof.

In grades nine through twelve matters are more debatable. Many mathematics teachers insist that algebra should be started in the ninth grade. This is particularly true in the large schools, where a considerable number of pupils might well take four years of mathematics. In those schools where only a year each of algebra and geometry are offered there is a strong argument for postponing these courses until the eleventh and twelfth grades. At that time the pupils are more mature, more able, and have better habits of study. And at this level fewer pupils who conspicuously lack the ability and interest to succeed would enroll. Especially if the pupils have previously had general mathematics, teachers can insist on better standards in grades eleven and twelve than in grades nine and ten. Also, and this is especially worthwhile, pupils who

go to college will retain a much larger part of their algebra and geometry if these subjects are taught in the later grades. There is, nevertheless, one limitation. If pupils have not had either algebra or general mathematics in grades nine and ten, then they will be handicapped in physics classes in their junior year. This situation, however, could easily be solved by postponing physics until the senior year.

6. *Who should take algebra and geometry?*

In many schools a rather vigorous campaign has been carried on to recruit pupils in algebra and geometry. This, unless some standards have been employed in guiding pupils with respect to algebra and geometry, naturally contributes to lower standards in instruction. This is especially the case because of the tendency for teachers to let inferior and mediocre pupils monopolize the teachers' time. In almost any school pupils ranking in the lowest third of mathematical ability, as determined by prognostic tests and arithmetic grades, should be advised not to take algebra and geometry until they have taken general mathematics, improved their backgrounds, and demonstrated their ability to succeed.

7. *What is the place of general mathematics in the schools?*

In grades seven and eight it is obvious that general mathematics has replaced pure and straight arithmetic. In high school in recent years the enrollments in general mathematics have steadily increased. It seems very likely that within a few years more pupils in the ninth grade will be enrolled in general mathematics than in algebra. Providing that the able pupils have been located and guided into algebra, this may well be an excellent development.

Two years of general mathematics should probably be offered in the larger schools, one in the ninth grade and another in the eleventh or the twelfth grade. In the later years, as the pupils more closely approach vocation and/or marriage, they will more easily recognize their needs for mathematical concepts and skills. Also

the applications and uses of mathematics then become more obvious to the pupils. Thus the pupils can be more easily motivated. They may understand mathematics better, moreover, when it is taught in connection with applications that are clear to the pupils.

Of course, both aptitude and future plans weigh heavily in deciding which mathematics to take. Since aptitude for algebra is less likely to be changed than the pupils' vocational plans, the election between algebra and general mathematics should be made more on the basis of aptitude than on ambitions.

8. *What is the place of vocational mathematics?*

This issue has been with us for many years. There is much to be said on either side of this dichotomous issue; possibly there will never be agreement on it. For those people going into the business world there may well be training in business mathematics. For those going into the skilled trades there should be a course in shop mathematics. For those going into agriculture there might well be profit in a course in agricultural mathematics. Perhaps, on the other hand, it might also be said that if pupils have had general mathematics there may be little need for them to take an entire year of vocational mathematics. Possibly one semester might suffice.

9. *Should mathematics beyond the eighth grade be required?*

The answer to this question depends to some extent on whether we are concerned with quality or quantity in our enrollments for courses. Even more important, however, is the nature of the required subjects.

There is no question that life in this world has been becoming more and more mathematical. For the last half century the needs for various types of mathematical training have greatly increased. A very strong case can be made for requiring one year of mathematics. A fairly strong case can be made for requiring two years of

mathematics. But if all pupils beyond the eighth grade are to be required to take mathematics, then the appropriate types of mathematics should be offered. And if an occasional pupil is conspicuously lacking in ability he might be excused from taking the second year of mathematics. Careful guidance is of course necessary, seeing to it that pupils make their choices in general mathematics on one hand and algebra and geometry on the other in the light of their abilities, their interests, their backgrounds, and their probable future needs.

10. *What should be the nature and amount of homework?*

The changes in modern life both within and outside the school have raised problems in the pattern of homework. With the increasing demands on the out-of-school time of pupils in both the elementary and secondary schools and with the increased opportunities for distinctly pleasurable activities, the problem of what should be the program with respect to homework has become a more serious and a more challenging one. It is clear that less reliance may be placed on homework for at least a considerable portion, if not the great majority, of young people. This argues for a somewhat longer school day and more opportunity for study at school.

There has been another kind of development in homework which will no doubt be employed by an increasing number of teachers. It is well described in a recent publication<sup>2</sup> of the National School Public Relations Association, a department of the National Education Association. Instead of having a considerable amount of general practice work at home, with which pupils may and frequently do receive assistance which does more harm than good, there is a tendency to use home projects as sources of information and data pertinent to the uses of arithmetic in life outside the school.

11. *Should mathematics be taught to give equal stress on its utilitarian values and its cultural potentialities?*

Should applications predominate? Or should the place of mathematics in the historical development of mankind, its place in present thought, its connection with logic, and its intrinsic interest as puzzle material also receive emphasis?

To most educators the answer probably is that, although utilitarian values have increased greatly in recent years, cultural potentialities should not be overlooked. This is especially important in classes made up predominantly of superior students.

<sup>2</sup> *It Starts in the Classroom* (The National School Public Relations Association, Washington, D.C., 1952), pp. 29-34.

## University of Virginia Institute

The second University of Virginia Institute for Mathematics Teachers will be held at Charlottesville, Virginia, August 2-13. It will be directed by Professors F. G. Lankford, Jr. and W. W. Rankin. This year the program of the institute is planned for elementary teachers of arithmetic as well as for high-school and college teachers of mathematics. Main features of the program include: (1) two series of daily lectures dealing with topics of practical concern to the classroom teacher. One series of lectures has been planned for elementary teachers and a second series for high-school teachers; (2) several evening lectures on applications of mathematics in industry, science, and government by persons

currently engaged in these fields; (3) study groups with daily meetings led by experienced teachers. Some of these are of especial interest to elementary teachers and some to high-school teachers; (4) demonstration teaching; and (5) abundant opportunity for a good time including a dinner in the Blue Ridge mountains, an informal tea, and a watermelon party.

Participants may enroll for credit or not as they wish. Two semester hours of graduate or undergraduate credit may be earned by those who elect to enroll for credit. Fees total \$28.00 for the two weeks. Inquiries should be addressed to: F. G. Lankford, Jr., 1-B West Range, Charlottesville, Virginia.

*Which way mathematics?*

## *Mathematics for our time*<sup>1</sup>

R. S. BURINGTON, *Chief Mathematician, Bureau of Ordnance (Navy), Washington, D.C., tells of the real need for better articulation between secondary schools and colleges in order that those who are able to prepare for technical and professional work, and those with other abilities, be adequately educated.*

TODAY WE are living in an age when a strong mathematics background is a necessity for many people. We are living in a rapidly changing age. In order to replenish and to provide for the present and future demands of our civilization, we must make provision for the strengthening and training in the mathematical sciences of many of our younger generation.

I should like to reflect on the meaning of this great use of, and need for, mathematics, in the light of the type of subject-matter many of our high-school and college students must master in order to keep pace with the times. They must master subject-matter in order to provide the proper background of knowledge and training in mathematics which our civilization requires now and will require in the years to come.

The great significance of mathematics in the present educational program has been brought forcibly to my attention on numerous occasions when my work has caused me to examine the many research, developmental, engineering, and production steps involved in producing a new or improved engineering system. I find that in all such cases mathematical anal-

yses or computations of some kind lie at the heart of the important steps in such programs. As a necessary, regular routine, tremendous uses are made of mathematical techniques in the majority of our great industries, both private and governmental. For example, in the aircraft industry, where hundreds of thousands of hours of advanced mathematical analysis and calculations are necessary for the design of a single airplane; in our great communications, electrical, and electronics industries, where almost nothing is accomplished which does not rest heavily and indispensably on rather advanced and extended mathematical techniques. This same situation prevails in the growing fields of nucleonics, in the huge chemical industries, and in many other diverse fields of business, production, development, and research. The list is too long to give here. More and more technical and business personnel in this country need better mathematical training in the early school years. While we are today witnessing the rapid development of great computing machines, they do not and can not think for us, nor will they ever be able to do so, not even for our children or our children's children. Such machines can not provide a substitute for, nor remove the necessity for, widespread training programs in mathematics and related topics. These great needs which are permanent—and

<sup>1</sup> Presented at Atlantic City, New Jersey, before The National Council of Teachers of Mathematics at their thirty-first annual convention. The opinions expressed herein are those of the author and may or may not reflect the official views of the Department of the Navy.

they are likely to increase—must be met by the early and adequate training of our young people in the high schools and the colleges of this country. The need for improved mathematical training is a steady one, and is not merely a transient one which some may feel is the result of the peculiar position of our country in the world situation today. We must treat this matter as a growing, continuing one and must make better provision for this need in our school systems.

As I have had occasion to observe from time to time, and as I reflect on the many years that I was privileged to teach both in secondary schools and in colleges and university graduate schools, I have noted that on the whole our teachers and our school systems have done and are doing a mighty fine job in training our young people for the future. They often have difficulty meeting all of the varying pressures that they must face in their work.

As I go about this great country of ours and observe the magnificent performance of our younger people—many working under severe handicaps brought on in part by World War II—I am impressed with the fact that our young people are beginning their careers too late in life. Their period of formal education is drawn out too long. This late entrance into their careers shortens their term of active professional life, lessens and inhibits their achievements, and reduces their usefulness accordingly.

Almost everywhere in this country, elementary and secondary education requires twelve years, and college at least four more years. As a result, few students graduate from college before they are twenty-two years of age, and those who serve in the armed forces seldom graduate before they are twenty-four years of age, if they graduate at all. All of this makes the long period of special training required today in so many professions especially hard on our young people, especially for those who must find time to devote several years to the service of their country. How

much better it would be if we could advance our capable boys and girls through high school by the time they are sixteen years of age, and through college by the age of nineteen years. How much better it would be for the country as a whole, as well as for these students, if they could enter their professions earlier than they now do. How much better it would be if they could experience earlier those environments and responsibilities which are so necessary for the early acquirement of maturity, and for the mastery of their respective professions.

As many of you realize, the typical mass education, which has evolved over the last two or three generations, has been developed for the good of the larger proportion of students endowed with what might be called typical ability. In most situations this educational program is probably not the best that we could provide for the coming generations, and in particular for those students with outstanding talents. The emphasis in most schools has tended toward helping the average students at the expense of retarding the more able students. This policy when followed tends to encourage mediocre performance and to inhibit the able. This policy also encourages poor pedagogy.

For the good of students of all types, of all potentialities, we must insure in so far as possible that those young people of real talent be given a chance to develop their latent powers early. Only in this way can the people as a whole profit most from the education of the many, and the few. Those with special talents among our young people should be given an opportunity to secure an education in order that they might serve their communities to the best of their potentialities. Unless we make special provisions to provide proper opportunities for these talented people, we cannot hope to provide the best for our future generations.

As I cast my eye over typical lists of high-school and college offerings I am impressed with the need for providing

special opportunities for these young boys and girls in the junior and senior high schools whose talents permit them to go much further and much more rapidly than the majority of their schoolmates. This need must be met in such a way as not to interfere with the development of these talented children as normal, growing people, who must lead normal lives with the rest of their friends and acquaintances.

Why cannot such talent be found and be permitted to progress through the first two years of a strong college course by the time they are seventeen or eighteen years old? Why should such talented students not study calculus and have thorough courses in physics, chemistry and biology before they are eighteen years of age, and why cannot they take these courses without any undue strain or warping of their personalities? I am sure that this can be done successfully, and without marring or spoiling the personalities of those in the program. I am sure that many young people of talent would actually show more interest in their accomplishments than they do now, were they not permitted to become so bored and disinterested with their elementary and secondary education. We actually lose much talent by following a course which breeds boredom and disinterest. The competitive spirit and a real challenge with opportunity for achievement will greatly improve things.

It is quite possible for young people to master ordinary plane and solid geometry in one year. Geometry can be taught in a far more interesting manner than is usual. It is possible to gain a good knowledge of college algebra and analytical geometry in our typical high schools, without in any way interfering with the normal development of the young people involved. Many of the elementary concepts of statistics and probability can be taught fairly early; many of the mathematical concepts of great importance and usefulness in the study of engineering and physical science, such as the concept of vector, derivative, integral, can be taught fairly early and

successfully; and in so doing, pave the way for a better and more thorough understanding of the sciences.

I urge that to meet the requirements of today and tomorrow, we must make it possible for our more talented young people to finish their ordinary mathematics through calculus before they are eighteen years of age, thereby advancing their college curriculum by roughly two years in almost all subjects. The beginnings of such a program can be made in our larger cities where the problem of providing for both the talented and the not so talented students should not be difficult. Both the high schools and the colleges can and must co-operate fully in setting up such programs. Such programs must permit both classes of students to enter college and progress rapidly and successfully, and without penalty, as a result of their more rapid secondary progress, or ordinary progress, as the case may be.

I have no doubt that many of you, if given the opportunity, would soon succeed in developing the details of the high-school courses in mathematics for such a plan. These courses could be made to include all of the advantages of repetition, drill, and repeated thought, but with progressively added difficulty in method. This could be done in such a manner as to produce intellectual inspiration and enthusiasm for the proper types of young people far beyond anything experienced today. Let us urge the school authorities in this country to institute such plans and courses in all of our larger communities and later in the smaller ones, beginning with our primary and junior high schools. Let us insist that the colleges co-operate fully so that there will be no penalty to those advanced students entering colleges by having to repeat needless course work which they have long since understood. Let us revamp our curricula to the point that a large portion of our talented young people have a good start in various fields, and in particular mathematics, before they are eighteen years of age. Early

mathematical training greatly simplifies the difficulties students have in mastering such subjects as chemistry, physics, engineering, and economics.

It appears that we might divide all secondary, and perhaps university and college, students into two categories of ability. In the first category are those who could progress through the secondary schools roughly two years sooner than they now do. In the second category are those who naturally follow a pace now commonly used in the secondary schools and colleges. Each of these categories is necessary to provide for the well-being of the nation in the future, but each has entirely different talents and potentialities.

Those boys and girls who belong in the first category exhibit a great interest and love to work, as long as what they are doing is in fact a real challenge or is something new to them. However, such people are not likely to be so interested in those things which they have learned to do well and which no longer present a challenge. In the second category, we find boys and girls who like to do those things which they have already learned to do, rather than accept the challenge of undertaking new things. This second category of student prefers to work at those things which he has already learned to do. This is quite a contrast to the tendency and behavior of students of the first class who prefer new challenges and new things to do, and who soon become bored with the things they have had before.

Plans for accelerating the education of talented young people have been followed in some form or another for a long time in Europe, and in certain schools and communities in this country. A program such as I have sketched would offer more stimulus and interest to the teachers than present procedures. If successfully followed, such a program would lead to the development of a larger percentage of more competent young people to assume

the burdens of leadership in the communities in which they may live; and, by their early and better training, they should prove to serve the communities even better than our leaders do today. Such a program is strictly a democratic one and is in no way discriminatory to any class. Certainly there are many details to be worked out, but I am sure that there is a successful solution.

If we are going to train the coming generations to take adequate care of themselves, and of subsequent generations, it is only proper that we make it possible to furnish the needed challenge for the first category students, without the handicaps that go with following the pace and content-matter at which the second category students prosper best. We should not waste the time and risk the lack of interest on the part of the more intelligent students in the first category, with slower and thinner programs designed for students of the second type. Neither should we hold the talented back, but rather we should make it possible for them to go at their natural pace.

Let us see to it that our competent and talented students progress while they are young and receptive, and at a rate commensurate with their natural endowments. Let us see that these students reach the level of professional maturity at an age when they can adapt themselves readily to all sorts of changes and difficulties and can assume their responsibilities as they should at an early age. Our secondary and college authorities, by co-operative efforts and decisive steps, must implement this program now. This must be done without imposing any handicap to either the students of the first type or those of the second type, each of whom must in time carry very important parts of the nation's responsibilities. This is necessary for the well-being of the individual, irrespective of his personal endowments, as well as for the continued well-being of the nation as a whole.

*Which way mathematics?*

## Elementary and secondary school training in mathematics<sup>1</sup>

S. S. CAIRNS, *Department of Mathematics, University of Illinois, Urbana, Illinois, originally presented his views in the October, 1953, issue of the American Mathematical Monthly. This article is reprinted here with the permission of the editor of the "Monthly." Dr. Cairns sets forth some shortcomings of the present program in mathematics and suggests remedial measures.*

### 1. INTRODUCTION

The general problem to which these comments pertain is that of adapting our public schools to the needs of society and the nation. Attention is restricted, however, primarily to the question of mathematical instruction, the field in which the writer is best qualified to offer advice. This is also a phase of the general problem to which a peculiar importance attaches as a result of the national emergency and the associated critical shortage of scientifically trained manpower. The shortage is expected to become worse before it gets better, since industry, the government, and the military services are making increasingly heavy demands with no signs of a corresponding increase in the supply. A substantial improvement in the situation could be effected by remedying some of the serious defects in elementary and secondary school mathematical training. It is the present object to support this assertion by discussing such defects and suggesting remedial measures.

<sup>1</sup> This is a revision of a statement presented to the School Problems Commission of the State of Illinois, meeting at Carbondale, Illinois, March 6, 1952.

### 2. THE PRINCIPAL QUESTIONS

Recent discussions of educational problems have raised subsidiary questions which becloud some of the main issues and present a danger to effective progress. These troublesome and almost irrelevant problems include (1) whether certain subjects are better taught now than at some previous time (2) who is to blame for some of the recognized shortcomings of our schools and (3) whether we are preparing most students to meet their expected needs in later life. We should rather concentrate on the magnitude and nature of our national needs, on the obstacles to meeting them and on methods for overcoming these obstacles.

### 3. MATHEMATICAL SHORTCOMINGS OF OUR SCHOOLS

To commence with generalizations, our high schools are sadly deficient both in preparing students for college and in offering adequate education to those not bound for college. Our elementary schools, in turn, are deficient in preparing students for high school.

Children of average to superior abilities, in the earliest grades, are frequently (per-

haps generally) offered no encouragement to proceed at their natural pace in learning those aspects of arithmetic which appeal to them. At a slightly later stage, they are introduced to the fundamental operations of addition, subtraction, multiplication, and division, but they are generally not drilled in such basic necessities as the multiplication tables. As a consequence they enter high school severely handicapped, save for that small proportion who learn so readily that they need no drill. In high school, the mathematics courses hit a slow pace, partly because the students have inadequate backgrounds, partly because there is no genuine incentive for the schools to provide suitable courses or for the students to take them. As a consequence, the colleges lose a year or two of mathematical training by having to teach to the freshmen, and frequently to the sophomores, courses which properly belong in the high schools. For students in the humanities, this situation is less serious than for those in the sciences and engineering. The latter are delayed to such an extent in commencing their essential studies that (1) a net loss of at least a year is difficult to avoid before their training is complete and (2) their programs are so crowded as to preclude many of the broadening studies which should form part of their general education. The delay carries over into the graduate schools, where we find ourselves teaching large proportions of our students material which belongs in a good undergraduate program.

Before substantiating the foregoing remarks, let it be noted that exceptional schools exist, generally in certain urban areas, and that in other schools exceptional teachers can be found who somehow partially counteract the general difficulties.

While a wealth of data could be offered in support of the preceding statement of shortcomings, a selection will be made, for convenience and brevity, from experience with two categories of students at the University of Illinois: (1) students

who enter the College of Liberal Arts and Sciences with deficiencies in mathematics and (2) students in the elementary schools program of the College of Education. There were 234 students in the first group in the period 1949 to 1951 and 268 in the second between 1947 and 1952. Deficiencies in mathematics imply only failure to have taken one full year each of high-school algebra and plane geometry. The lack may be due to lack of opportunity or to a deliberate avoidance of the subject for one reason or another. There is evidence that a good proportion of the students with such deficiencies (constituting about 10 per cent of the L.A.S. freshmen) are suitable, though poorly prepared, college material. Identical standardized arithmetic tests have been administered to both groups. The results reveal a shocking inability to handle elementary arithmetic. To mention a few examples from data supplied by Mr. Clarence Phillips of the Mathematics Department, only 41 per cent of the first group and 59 per cent of the second correctly figured one year's interest at 6% on \$175; the percentages of success were 34 and 55 in computing  $7-6+2-4$ , and 30 per cent and 53 per cent in arranging the numbers .40, 2.5 and .875 in order of magnitude. The difference between the two groups is due to the fact that the second group (1) had more mathematics in high school, (2) is more selective as to admission, and (3) contains 77 per cent seniors and graduate students, while the first group is almost all freshmen.

Of the students entering with mathematical deficiencies 50 per cent fail to attain sophomore standing. This percentage is far out of line with the native abilities within the group and reveals the handicap of a student who is so poorly prepared by his high school.

The College of Education students just mentioned are required to take an arithmetic course in the Mathematics Department, intended to deepen their understanding of what they will soon be teach-

ing. Many of them are deplorably weak in the fundamentals, will hesitate over such things as eight times seven or seven plus six, and will frankly express their easily understood fear of teaching arithmetic. This often takes place in the second semester of the senior year, after they have done practice teaching and a few months before they will be on the job, perhaps unconsciously transmitting their own aversions and lack of confidence to their students.

Turning to the College of Engineering, suffice it to remark that the inadequate mathematical preparation of the entering freshmen recently led to the establishment of a joint committee from that College, the College of Education and the Department of Mathematics. The work of this committee culminated in a pamphlet entitled *Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois*,<sup>2</sup> which has been widely circulated among Illinois high schools. At present, a similarly composed committee, under the chairmanship of one of our University High School mathematics teachers, is studying means of adjusting the high-school program to meet these needs. This co-operative effort is encouraging. It is to be hoped that the work of the committee will lead to widespread improvements in the teaching of mathematics and will serve as a model for co-operation elsewhere.

#### 4. UNDERLYING CAUSES FOR SHORTCOMINGS

This partly speculative section could run to great lengths. To avoid that, a few false principles will be listed, with brief comments and with no effort to estimate how widely these principles are accepted by those responsible for administering our schools.

##### a. *The theory that drill and deliberate*

<sup>2</sup> *Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois* (Office of Publications, University of Illinois, Urbana, 1951), Vol. 49, Number 9.

*memorizing must be avoided, especially in the lower grades.*<sup>3</sup> This clearly works to the detriment of (1) learning the multiplication tables, (2) learning the alphabet at the proper time, (3) learning to spell, (4) learning the essentials of English grammar, (5) at a somewhat later stage developing a vocabulary when studying a foreign language, and (6) acquiring the study habits demanded by effective college work. This theory is generally associated with the unwarranted belief that techniques will be incidentally acquired.

- b. *The theory that local needs and desires should dominate in determining curricula, to the practical exclusion of needs on a national scale.*
- c. *The theory that high schools should limit their programs to those skills and manipulations that some group of individuals finds necessary to the average adult.*
- d. *The belief that the less successful students should be kept in the same classes with the more successful.*
- e. *The aversion to competition among students.* This, and the previous item have a deadening effect on those who should be stimulated and encouraged.

#### 5. A PROPOSED GUIDING PRINCIPLE

To quote from a letter by Professor J. W. Peters of the Department of Mathematics at the University of Illinois, "The American public high school has the responsibility to develop and administer

<sup>3</sup> This footnote is being added on the occasion of the reprinting of the present article in *THE MATHEMATICS TEACHER*. Several people have pointed out that Section 4a might create the illusion that the author advocates drill and memorizing independently of understanding. On the contrary, he favors well-motivated drill and believes it possible to supply acceptable motivations in all cases where drill and memorizing are important. For example, if a classroom (Heaven help the teacher!) contains seven rows of eight students each, then a student can easily appreciate the advantages of knowing eight times seven as compared with having to count the students in the room.

a sound educational program which will provide for the education at the high school level of every youth to the full extent of his capacity and ability." This is a lofty ideal, but one which we can take as a guide, even though its full realization may be in the distant future.

#### 6. POSSIBLE REMEDIAL MEASURES

The following suggestions were offered for the consideration of the School Problems Commission:

a. The establishment of standards on a state-wide basis for the grade schools and high schools. The enforcement of such standards would require some sort of testing procedures. The merits of the Regents Examinations of the State of New York might be considered in this connection. In the establishment of standards and indeed in all phases of studying school problems, it is essential that due consideration be given to the views of scholars and scientists as well as to those of Departments and Colleges of Education, parents, industrial employers, and educational administrators.

b. The establishment of adequate college entrance requirements. This would involve upward revision, which could be introduced only gradually, as elementary and secondary schools adapt themselves thereto. In this connection, certain quotations may be in order from Bulletin Number 9 of the Illinois Curriculum Program series, entitled *New College Admission Requirements Recommended*, issued through the office of the State Superintendent of Public Instruction.

"The specification by the colleges of cer-

tain high school courses to be taken by all students seeking college entrance sets definite limitations to curriculum revision. If a considerable block of courses must be retained in the high school to provide for the preparation of students who hope to go to college, the opportunity to re-examine the total high school curriculum and to replan the program in terms of the needs of all high school youth is hereby curtailed." (p. 5) This disturbing quotation suggests that no essential college preparatory courses are recognized on the basis of all our educational experience to date.

"The committee recognizes that small high schools will not always be able to provide a sufficient variety of specialized courses to meet the need for the special programs of all its graduates. In such cases, the colleges are urged to make provision for the basic specialized work with as little handicap to the student as possible." (p. 14) This implies that the colleges are to continue, as a matter of policy, and perhaps even to expand, their offerings of high-school types of instruction.

"With limited resources, the high school's first responsibility is to provide education of general value to all its students, rather than to provide for the specialized needs of parts of the student body when the latter effort is taken at the expense of a good program of general education." (p. 13) This is at variance with the guiding principle suggested above. It implies that, although talent is uniformly distributed throughout the population, students in certain communities are to be denied the opportunity for their full development.

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### Workshop in Secondary Mathematics

IOWA STATE TEACHERS COLLEGE  
CEDAR FALLS, IOWA  
June 14-25, 1954

The theme is *Classroom Practices to Fit Modern Theories of Instruction*.

The teachers bring their problems with them; they examine statements like those in the Twenty-first Yearbook of NCTM; then they

plan teaching units for 1955 in the light of modern theories of instruction.

Resources are the curriculum laboratory, the college library, the mathematics laboratory, and samples of teaching aids. The staffs of the college and the campus school are available on call. Visiting consultants from off-campus are Kenneth Henderson (Illinois) and Paul Trump (Wisconsin). The co-ordinator is Harold Trimble.

## *Which way precollege mathematics?*<sup>1</sup>

KENNETH O. MAY, *Carleton College, Northfield, Minnesota.*

*What type of drill should be used in mathematics classes?*

*Can the nature of proof be taught in elementary algebra?*

*What is the function of the problem in mathematics?*

It is encouraging that many mathematics teachers, in both high schools and colleges, are dissatisfied with mathematical education at all levels, and that they are searching for ways to bring it in line with the increasing demands of the times and with the dramatic development of mathematics in this century. In spite of long hours, large classes, low salaries, distracting extracurricular activities, disciplinary problems, the legitimate needs of the majority who are not going to college, and the general anti-intellectual atmosphere, high-school mathematics teachers are doing a commendable job of preparing pre-college students. In the words of C. V. Newsom, formerly chairman of the Mathematics Department at Oberlin College and now an associate commissioner of higher education of New York, "In spite of widespread criticism of our high schools, there appears to be little actual evidence that they are providing precollege training now that is inferior to that of former years."<sup>2</sup> Data of the kind presented by Professor Cairns show merely that college students who have taken less than the minimum high-school mathematics and those who have been many years without any mathematical instruction are very weak in

arithmetic. That 41 per cent of the weakest 10 per cent of entering L.A.S. freshmen at Illinois appear to know the meaning of "6%" is a tribute to our secondary teachers and a striking proof that their efforts are not entirely wasted, even on the most "hopeless" of their students.

The present crisis in mathematics is due not to any deterioration in the work of mathematics teachers, but to an urgent national need for more and better mathematics at a time when administrators and the public have for years slighted mathematics and, indeed, discouraged all vigorous mental effort in the high schools. The high-school program seems better suited to meet an urgent shortage of athletes and club-joiners than of scientists and other professionals. In athletics and other "co-curricular" activities, there are numerous awards and keen competition for them. In contrast, excellence in scholarship is permitted, but mediocrity is considered more "democratic." Our high schools, while claiming to prepare students for real life, have established a system of status and rewards that is just the reverse of that in the world. When school is over, income and status for most people depend on how they do on the job, whereas in school success is made to depend on how they do in recreational activities. The contrast is especially sharp for precollege students, whose future is determined primarily by the soundness of their understanding of

<sup>1</sup> Thanks are due my colleague Professor Kenneth Wegner for several helpful criticisms of the first draft of this article.

<sup>2</sup> Quoted from an address to the Missouri Section of the Mathematical Association of America, April, 1953.

subject-matter and only secondarily by their extracurricular experiences. It is high time that scholarship be appropriately recognized, publicized, and rewarded as the main job of students. Athletic and other extracurricular activities should be given a subordinate place as essential parts of a rounded education.

One reflection of the existing inversion of values is the opposition to grouping students by ability or to marking them competitively. No one thinks for one moment of abolishing ability grouping for football players or high-school debaters, because it is obvious that competition as well as co-operation are required to produce the best in these fields. No one sheds tears for those who don't make the teams, and no one argues that the poorer players would benefit from an indiscriminate mixing. That entirely different reasoning is applied in the field of scholarship reflects the fact that administrators are not as anxious to turn out good scholars as they are to have winning teams. But from the national point of view, it is of no importance that high-school students excel in athletics. The important thing is merely that they participate in them. In contrast, it is of supreme importance that every young person do his very best in his studies. This can hardly be expected when incentives and rewards all pull the other way.

That teachers have managed, on the whole, to maintain standards under these conditions is remarkable. But the fact is that what was done in former years is not good enough today. We need both a quantitative increase (more mathematics taught to more students) and a qualitative improvement (new and better mathematics taught more effectively). The quantitative increase can be assured only by energetic efforts to convince administrators, counselors, and students of the true practical and cultural importance of mathematics. The improvement in quality depends upon a whole series of measures. Perhaps the most urgent (and least

likely) are improved working conditions so that teachers have the time, energy, and funds for self-improvement. Fellowships, subject-oriented summer institutes, and suitable summer courses are essential. State-wide and nation-wide standards (provided they are set up co-operatively and constantly modified with the times) and adequate college entrance requirements (provided that they are kept up to date so as to stimulate rather than block improvements) are certainly desirable. Above all, we need working co-operation between high-school and college mathematics teachers directed toward a continued improvement of the mathematics curriculum at all levels.

This writer feels strongly that our needs will not be met merely by more energetic drilling of students in the traditional list of manipulations in algebra, geometry, and trigonometry. At its best, this kind of teaching produces a student who, like a Pavlovian dog, responds as expected to the approved list of questions. But usually he has learned his tricks without understanding and gets along only if nothing is demanded but the old tricks or the learning of new ones. Such students run into difficulties as soon as understanding and thought are required.<sup>3</sup> If this does not oc-

<sup>3</sup> The following quotations are from responses of above-average freshman mathematics students to an unsigned questionnaire concerning their high-school and college mathematics work. "I was given a formula and told how to make the substitutions, and then I was expected to add and subtract right. College has more logic instead of just working the problems. I have never done this and so I am completely lost." "In high school a few rules were given with which one merely 'ground out' answers." "In high school I think too much emphasis is on doing lists of problems almost like copy work. In college we have learned more of the whys and theory, and the math learned here I think is easier to apply." "The more methods you could commit to memory the better." "... automaton-like work..." "... we went through arithmetic and algebraic processes without any thought of why these things were so. We did very little if any proving and arranging our thoughts in a logical order..." "... blind memorization of laws..." "High-school mathematics concentrated on the mechanical operation and the answer..." Apparently "drill and deliberate memorization" are still very much alive in our high schools. The trouble seems to be that they are too seldom combined with other essentials of training. Of course, these comments apply equally well to much of present college teaching.

cur while the victim is still able to learn to understand and use mathematics, he simply concludes that it is "impractical." And he is quite right concerning the "mathematics" he has known. At worst, this traditional drillmaster kind of teaching produces a lifelong distaste for mathematics and a conviction that it consists in a sterile collection of rules of thumb for doing unpleasant calculations. There is no doubt that such attitudes underlie the neglect of mathematics during the last few decades.

In spite of the widespread suspicion of "general mathematics" and the belief that it represents merely a lowering of standards, I am inclined to think that it has been a real step forward in educating the majority that do not intend to go to college. Undoubtedly its content should be strengthened, but it has the merit of presenting mathematics as a single body of living knowledge, closely related to every aspect of our culture. Moreover, the authors of general mathematics texts have been pushed toward concentrating on fundamentals, and they sometimes achieve greater mathematical soundness than the traditional sequence texts. Of course, the general mathematics courses are far too thin for college preparation, but, in my opinion, we have much to learn from the experimentation in general mathematics and other courses for the noncollege group.

During recent years teachers have been quite right to resist changes in the traditional precollege sequence when it was a question of lowering standards. However, it is now becoming clear to many that the curriculum that traditionally begins with elementary algebra in the ninth grade and ends with integral calculus in the college sophomore year is due for a drastic overhauling. There is a growing body of informed opinion that the entire precollege and college program in mathematics is due for complete revision, involving the replacement of much traditional material and a completely new sequential arrange-

ment.<sup>4</sup> It is not a question of lowering standards, but of changing and raising them.

The new curriculum should ignore the traditional sequence and present mathematics as a single subject, in the order dictated by sound psychology and by the logical structure of the subject in the twentieth rather than the seventeenth century. It should discard something like half the traditional manipulative topics, which have been aptly described as "ideally suited for solving engineering problems of 1915," but which are today outmoded, impractical, and mathematically trivial. It should concentrate attention on fundamental ideas and laws that are applicable throughout mathematics, science, and all thoughtful activity. It should bring into the high school a number of topics (such as elementary calculus) that have in the United States (though not in any other country) been traditionally postponed to college. It should include some twentieth century mathematical ideas (such as set theory, modern statistical inference, and symbolic logic) that are both elementary and practical. It should develop manipulative skills by accustoming students to apply basic principles in a very wide variety of situations. Its main goal should be to enlarge the ability to use mathematical ideas in formulating and solving unfamiliar problems, in making and checking guesses, in proving or disproving claims, and in deducing or finding information. It short, it should concentrate on teaching young people to think mathematically rather than on teaching rules for solving special problems.

<sup>4</sup> Such conclusions have been reached independently by three national committees of high-school and college mathematicians during the last year. They are the Mathematics Committee of the School and College Study for Admission with Advanced Standing, chaired by Prof. H. W. Brinkman of Swarthmore, the Joint N.C.T.M.-M.A.A. Committee on the Training of High-School Teachers, chaired by Dr. C. V. Newsom, and the M.A.A. Committee on the Undergraduate Mathematics Program, chaired by Prof. W. L. Duren of Tulane. Several other groups and committees have expressed similar views.

The previous paragraph may seem utopian, yet many of its ideas are quite current in colleges and high schools, and some steps have been taken in the indicated direction. For example, much of what is traditionally called "analytic geometry" is now taught as an integral part of "algebra," and in some schools plane and solid geometry are integrated to the benefit of both subjects. There is no doubt that experiment and change will continue, and I think it will be toward the kind of curriculum suggested, because only such a program will meet the need for well-trained people. Since space does not permit discussing this further here, I shall mention briefly a few controversial matters upon which action can be taken by every teacher without waiting for new texts or major changes in policy.

1. *Drill.* The question "Should we have drill?" appears to ignore the fact that students are inevitably drilled day after day in our classes. The only question is "What kind of drill?" Is it to be the thoughtless, automatic drill of the parade ground, now found to be unsuitable to modern warfare? Is it to be drill in laziness and mediocrity? Or is it to be drill in alertness, independent thinking, questioning of assumptions, and energetic attack on problems? If it is to be the latter, more thinking and effort is required of both student and teacher, and this can be expected only if strong incentives are provided for both. Blind drill in thoughtless manipulations and drill in effortless "life-adjustment" are, in my opinion, equally inadequate substitutes for education.

2. *Proofs.* Proofs are often omitted on the ground that theorems can be "learned anyway" or that proofs are "not practical." Of course, blindly memorized proofs are as useless as blindly memorized theorems or methods of solving problems, and it is certainly not possible to prove every theorem that is used. But to ignore proof is to miss the whole significance of mathematics and science. Most of the theorems of Euclid were known before his

time. His great achievement, which has served as a pattern for the organization of all scientific knowledge, was to see that the limitless number of isolated geometrical facts are the logical consequences of a very few simple assumptions. The student who grasps this is on his way to understanding science, and has at his disposal a powerful tool for organizing his own knowledge. He no longer has to rely solely on his memory, but knows that things can be "figured out" logically provided a few basic ideas are understood.

The memory of any particular proof is of little importance, but the understanding of the nature of proof and experience in constructing and explaining proofs are valuable by even the most narrow-minded "practical" standards. Logical deduction and observation are twin activities upon which our whole civilization rests, and an early acquaintance with them both is essential for anyone who is going to make any significant contribution in science or technology. I would much rather have a freshman student with experience in deductive thinking but without the memory of a single theorem of geometry, than one who knew ten thousand theorems but had no idea of the nature or importance of reasoning. Instead of slighting proof, we should extend it to elementary algebra, in which many proofs are far easier than the traditional demonstrations of Euclidean geometry. Training in logic should be combined with all mathematical teaching for another, profoundly important reason. Clear thinking on the part of every citizen is the nation's only effective defense against the persuasive illogic of the enemies of democracy.

3. *Problems.* The importance of problems is that they provide practice in applying fundamental principles, thus preparing the student for the real problems he will meet later. Problem-solving should be approached as a matter of experimentation, guessing, checking, and reasoning. It should always be a question of looking for something—sometimes a nu-

merical answer, more often a method, a proof, a counter-example, or a new and unexpected idea. Problems that can be solved merely by "plugging in," when done in moderation, serve the useful purpose of familiarizing the student with new material and maintaining good computational habits. But problems of this kind grossly misrepresent the way in which mathematics is constructed and used in real life. A human being, rather than a machine, is needed to solve a problem only when something more than "plugging in" is required—namely creative and original thinking, even if of only a very simple kind. Drill in problem-solving that does not involve student initiative is not only a waste of time (except to prepare students to pass tests consisting of such problems), but it develops the harmful habit of being unwilling to try to solve a problem unless the correct method is already known. In advanced mathematics, science, engineering, and business, only

trivial problems can be solved by people who are unwilling to take a chance. The main goal in solving problems should be to encourage students to combine experiment and guessing with sound deduction and computation.

In this short article, it has been possible only to make a few, largely unsupported, and possibly exaggerated assertions. It is my hope that these will stimulate further discussion between high-school and college teachers. In the last analysis, every criticism of the high-school program is a criticism of the colleges, which, after all, have trained every teacher and administrator in the secondary system. The fact is that college and university mathematicians have largely ignored their responsibilities to primary and secondary education. We in the college field have, I think, as much to learn from our high-school colleagues as they have to learn from us, and a close working collaboration is long overdue.

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## The Mathematics Institute

*Sponsored by*

THE ASSOCIATION OF TEACHERS OF MATHEMATICS  
IN NEW ENGLAND

The Sixth Annual Institute for Teachers and Professors of Mathematics sponsored by the Association of Teachers of Mathematics in New England will be held August 19–24, 1954, at the Massachusetts Institute of Technology, which lies along the Charles River Basin in America's Trillion Dollar Row, just across the Harvard Bridge from Boston. The April issue of *THE MATHEMATICS TEACHER* published the tentative program. The Central Committee is now ready to announce the following additional discussion groups: "The Story of the Development of Mathematics up through Calculus" to be led by Mr. W. W. Rankin, Professor Emeritus from Duke University; "Teaching for Transfer in Mathematics" to be led by Mr. John J. Kinsella, Chairman, Department of Mathematics Education, College of Education, New York University; a series of five discussions especially planned for supervisors and teachers in elementary and junior-high schools to be led by Mr. Lee E. Boyer, Head, Department of Mathematics, State Teachers College, Millersville, Pennsylvania; "Mathematics for the Sec-

ondary School" to be led by William David Reeve.

For a complete program write to Miss Esther Emerson, 931 Broadway, Haverhill, Mass. To be sure of a room, register before June 10. *Mrs. M. Isabelle Savides, General Chairman.*

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## Call for Nominations

The nominating committee is calling for one or more nominations for each of the following offices: Vice President, College level; Vice President, Junior High School level; and three members for the Board of Directors. Send nominations before August 1, 1954 to the Chairman.

The members of the nominating committee are:

Jackson Adkins  
Clifford Bell  
Harry Charlesworth  
Maurice Hartung  
Houston T. Karnes  
Mary Potter  
Mary Rogers  
C. V. Newson  
Agnes Herbert, Chairman, 806 East North  
Avenue, Baltimore 2, Maryland

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# Mathematics for life adjustment

LAURA BLANK, *Hughes High School, Cincinnati, Ohio*, shows how mathematics is needed for normal adjustment to life situations.

THE GOAL of life adjustment education is to increase the effectiveness of present education efforts to meet the imperative needs of all youth.<sup>1</sup> The subject of this paper is mathematics and life adjustment education. Is it possible to plan and to develop new courses in mathematics or is it possible to make our courses "more realistic within the existing framework, *i.e.*, the objectives, content, and experience can be changed without altering the title of the course."<sup>2</sup> An implication of the quotation is that changes might be introduced without changing the basic philosophy of teaching mathematics.

Active and busy classroom teachers of high-school mathematics would welcome the production of new textbooks for the ninth or tenth grades and the eleventh or twelfth grades that are not merely reviews of arithmetic or refresher mathematics. These textbooks ought to treat subjects and problems of current life adjustment on the maturity level of a secondary-school student. In the Cincinnati school system, committees are at work on units in mathematics for a life adjustment program. However, the committee members find little time or energy for accomplishing their assignments while carrying a full teaching load.

Many textbooks do not get written because teachers cannot afford to take a year away from teaching to write. If

teachers wait until retirement to write a textbook, they often find the zest for writing has waned. Moreover, publishers are not as interested in a manuscript written by an instructor *emeritus* as in one written by an instructor with active, professional affiliations. If excellent teachers could live two concurrent lives, one for teaching to test out theories concerning the organization of mathematics materials and the other for writing, there might appear more new textbooks! The wish for textbooks to serve the needs of noncollege preparatory students exists throughout the United States. Most of the textbooks offered by publishers for these students are dull reviews of elementary mathematics, containing little material to serve as motivation for an adolescent.

A wise procedure to follow is to scrutinize carefully the subject matter of the present high-school courses to see where emphases can be changed or applications introduced. Such a procedure may lead to better life adjustment through education. Consider the first-year algebra course. Word problems concerning digits and the consequences of reversing digits in numbers should be abandoned. Age problems and coin problems fall in this same category of "puzzle problems." The only justification for problems of this sort is that they may train students in analysis of word statements and in choosing effective symbols for solution. Parents find little interest in helping with homework when assignments contain such problems.

Teachers spend too much time on exponents and radicals. Purely mechanical work with the manipulation of exponents

<sup>1</sup> "Life Adjustment Education for Every Youth" (Washington: Federal Security Agency, Office of Education, 1948), p. 4.

<sup>2</sup> Richard A. Mumma, "Are College Entrance Requirements an Obstacle to the Development of a Program of Life Adjustment?" *National Association of Secondary School Principals Bulletin*, XXXIV (May 1950), 162.

and radicals is not instructive to students. More time should be spent on developing a concept of an exponent or a radical.

On the other hand, let us have students estimate results whenever possible. Estimation of a result should be done *before* a problem is solved. Arithmetic exercises, problems of application of geometry facts, trigonometry problems, and slide-rule exercises offer many opportunities for estimation of results. A student should understand that a number obtained by measurement is an approximate number. He should be familiar with the fundamental rules concerning computation with approximate numbers.

At the present time, during a period of many gadgets and much computation, a slide rule is an invaluable instrument. Ninth-grade students can be taught to solve correctly simple exercises on a slide rule, even if the instruction must be by rule. Third- or fourth-year students, after some instruction on logarithms, can learn to use a slide rule with understanding; such students learn to appreciate the usefulness of a slide rule in solving problems in mathematics and science. They realize the limits of accuracy of a slide rule and discover applications for it outside a classroom. For example, slide rules are used extensively in scientific research, industrial production, the armed forces, and in navigation.

Ratio and proportion problems can be easily solved by means of a slide rule. There are many applications of ratio and proportion in the graphic arts, home economics, and the industrial arts. To read and interpret a blueprint requires a knowledge of ratio and proportion. There are many problems that use the concepts of ratio and proportion in family budgets, savings, discounts, commissions, loans, and interest rates. Boy Scouts use proportion to solve some of their problems.

The trigonometry of the right triangle furnishes an interesting and valuable use of ratio and proportion. Certainly it is important to teach every boy and girl the generalization represented by Pythagoras'

theorem, as well as the special case of the 3-4-5 triangle. There are many opportunities to show applications of Pythagoras' theorem in ordinary home living as well as in industry.

A first course in algebra should contain many applications of percentage. Some applications have been mentioned above, but emphasis should be given to the concept of percentage as it applies to life situations. Even girls are sports minded enough to be interested in checking or developing batting averages. Another application of percentage lies in the problem of taxes. Income tax forms, with their percentages of gross income and their tables for computing a final tax, furnish a rich source of real problems.

Some elementary geography has mathematical implications. Why do time zones on the earth exist? Why is each time zone approximately  $15^\circ$ ? To the uninitiated, it is startling to read in a commercial airline timetable that one can take off from Cincinnati at 5 p.m. and arrive at Indianapolis at 5 p.m. the same afternoon. Many complications and difficulties in travel are a result of failure to understand time zones and daylight-saving time.

Although the vocabulary of algebra is technical, it is not so involved that there exist no elementary applications. A student will be able to adjust better to life situations if he understands and can use the language of algebra concerning relationships, constants, variables, and functions. Such words as "formula," "postulate," "axiom," "theorem," and "hypothesis" are used in all areas of expression. An understanding of these words (and others) should be a part of life adjustment education in mathematics.

A function can be expressed by a statement, a formula, a graph, or a table of related values. Perhaps the local Red Cross chapter expresses the success of its drive for memberships and contributions as a graph. Should not all students who have had the advantage of a secondary-school education understand such a visual means of presenting data? Newspapers,

periodicals, and advertisements use graphs in one form or another. No one can really understand these uses of graphs without having drawn and studied each of the various sorts of graphs.

Tables of related values should be studied for understanding of the relationships involved. How does one variable behave as a second changes? If one variable increases, how does the other variable change? Could a better concept of the relationship existing between the variables be obtained from a graph? Or a formula?

An important formula in automobile driver education is the braking distance formula  $d = 0.055r^2$ , where  $d$  is expressed in feet and  $r$  is expressed in miles per hour. If a high-school student can understand this formula, it cannot fail to help him realize its implications. Every element in the formula should be carefully described and discussed until students understand the relationship expressed. Then it is time to study the implications of the formula for safe driving. Make a table of related values by assigning values to  $r$ .

TABLE OF BRAKING DISTANCES	
Speed in mi/hr	20 30 40 50 60 70 80
Braking distance in feet	22 49 .5 88 137 .5 198 269 .5 352

Another element to consider in safe driving is the reaction time distance. Reaction time distance is the distance a car travels from the point where a driver first "sees" danger to the point where he gets his foot on the brake pedal. Students quickly arrive at the conclusion that reaction time distance is a function of reaction time and speed. Further discussion or research or experiment can establish that reaction time varies with individuals, being between  $\frac{3}{4}$  sec. and 1 sec. Hence, reaction time distance varies from  $3\frac{1}{2}$  ft. to  $117\frac{1}{2}$  ft. as the speed varies from 20 mi/hr to 80 mi/hr. Reaction time distance must be added to braking distance in order to secure the total stopping distance for an automobile at a given speed.

It is the hope of mathematics teachers in the Cincinnati school system to reorganize mathematics instruction so that normal students will adjust better to life situations. By encouraging classroom innovations, by co-ordinating efforts, and by publication of results it is hoped that more teachers will be stimulated to action. This paper makes some suggestions of material that can be used in a first-year algebra course. The *concepts* are the same as in the usual algebra course, but the *means* for developing these concepts have as their sources contemporary applications.

### Northwestern University Workshop

A Workshop on Mathematics Education will be held on the Chicago campus of Northwestern University from August 2-20, 1954. The purpose of this workshop is to provide teachers as well as representatives of school systems an opportunity to make a study of specific problems in the teaching of mathematics which are of special concern to teachers and school administrators. Members of the workshop will be able to devote a three-week period to the examination of research, materials of instruction, and classroom practices pertinent to the con-

sideration of practical, suggested solutions to curriculum and instructional problems in elementary schools, and junior and senior high schools. Group studies will be devoted to such areas of mathematics education as curriculum, psychology of learning, mathematics needs in vocations and occupations, the retarded child, the gifted student, evaluation, instructional and learning aids, and minimum equipment needs of the classroom. Further information may be obtained by writing to E. H. C. Hildebrandt, 221 Lunt Building, Northwestern University, Evanston, Illinois.

# The mathematics teachers' opportunities for guidance

KENNETH E. BROWN, *U.S. Office of Education, Washington, D.C.*,  
*points out that "the primary task of guidance lies with the classroom teacher." He also tells what many classroom teachers are doing about guidance in the classroom.*

THE FUNDAMENTAL PROBLEM in guidance is to help the pupil discover and develop his desirable potentialities. This implies that, first, the basic needs and pattern of the pupil be understood and, second, the particular environment be provided that will best nurture his development.

Some pupils fail courses in mathematics because they were guided into the courses without a proper consideration of their previous experiences in mathematics. For example, if the pupil has developed a hatred for mathematics in the eighth grade, his attitude may affect his achievement in the ninth grade; or if a pupil has failed to understand fractions in arithmetic, it is unlikely that he will readily understand fractions in algebra. The reason for his lack of success in understanding fractions may be due to poor mental or physical health, undesirable teaching methods or lack of potential in mathematics. At times the overworked teacher with large classes is inclined to feel that most of the pupil failures are due to the last factor. However, we must admit that few pupils work at their maximum capacity or realize their full potential in mathematics. No doubt there are many failures among pupils who listen to explanations and struggle with exercises for which they do not have the prerequisite understandings. In any case, the pupil failure may be decreased under intelligent guidance.

Other pupils discover when their high-

school career is nearly completed or in their early college experience that they should have taken certain courses in mathematics. This is evidenced by the thousands of pupils each year who take noncredit courses in mathematics during their first year in college. No doubt many other capable pupils are lost to valuable scientific careers because they do not have or use the opportunity to take basic make-up courses. These pupils were not given the proper educational environment to develop them for their college experience.

Teachers and guidance counselors are not attempting to control the life of a pupil when they seek information and make suggestions. The purpose is to guide the pupil to information and give him experiences that will serve as an adequate basis upon which he can make intelligent decisions. To the extent that his decisions cause him to develop of maximum worth to society and himself, to that extent their guidance is a success.

## GUIDANCE ACTIVITIES OF THE ADMINISTRATORS

The administrators of public schools are attempting to provide better guidance in many ways. Guidance counselors are provided whose duty it is to co-ordinate the guidance services and furnish technical assistance. School administrators encourage and make possible such activities as assembly programs that furnish

vocational information, planned field trips, exploratory courses in the beginning years of high school, and school health clinics that gather important data. They provide special classes such as mental hygiene, marriage, personal living,\* and consumer mathematics. Also provision is made for classes necessary for the development of special groups of pupils such as classes in industrial arts, advanced mathematics, electricity, and child care.

Many administrators are contributing to the guidance program by encouraging the maintenance of a cumulative record system for each pupil. To be most helpful the record should reflect the pertinent characteristics of the pupil. The data usually indicate the pupil's achievement marks throughout his school experience, recent achievement and aptitude test scores, hobbies, attitudes, physical weaknesses and strengths, anecdotal records by former teachers and pertinent information about the pupil's home life. With these data available, the guidance counselor and teacher can be of more help to the pupil whether it is in planning a program of studies or social activities.

#### GUIDANCE ACTIVITIES BY THE CLASSROOM TEACHER

Although it is desirable, and in many cases necessary, for effective guidance to have the full support of the administration, the primary task of guidance lies with the classroom teacher. The pupil's greatest contact with the school is through the teacher.

If the mathematics teacher is typical of the 29,000 teachers in the United States who are devoting full time to mathematics, he has five classes daily with approximately 35 pupils in each class. He has 175 living opportunities for guidance each day. The large number of pupils per teacher permits only brief association with individual persons. This fact in itself stimulates the teacher to make the most of every guidance opportunity. Many teachers are assisting pupils in planning activities that

will give them an insight into many areas of learning. These experiences serve as a basis for making intelligent decisions about later life problems.

For the pupil to have equal opportunity with his classmates in achieving maximum development it is necessary that the teacher know his educational abilities, achievement, and environment. Teachers often try to secure this information through pupil class activities, tests, out-of-class activities, a study of the community, and an acquaintance with the pupil's family. The more pertinent information the teacher possesses, the better position he is in to advise and help pupils arrive at intelligent decisions. In addition to these routine duties, what other things are classroom teachers doing in guidance?

1. They encourage the use of the method of the scientist. For example, the projects proposed may consist of the collection of data and the interpretation of the data in the language of mathematics.
2. They encourage pupils to read books and articles that reveal the use of mathematics in careers where mathematics is essential such as those of scientists, engineers, and research workers.
3. They display career information on the bulletin board that produces an awareness of the need for mathematics in many occupations.
4. They show films of opportunities and requirements for careers in mathematics and its related fields.
5. They keep a current file of career information available for the use of pupils.
6. They encourage the pupils to explore opportunities and requirements of careers that are revealed in the study of the applications of mathematics.
7. They encourage superior students to enter mathematics contests and take part in mathematics assemblies, fairs, and exhibits.

8. They provide pupils with pamphlets, charts, and articles that show the growing importance and place of mathematics in skilled labor and business careers.
9. They encourage pupils to study in small groups the mathematics suitable to their ability and needs. All groups may be studying the same topic such as the formula with one group studying the relationships expressed in formulas found in home economics, another group the formulas in business journals, while another group is confining its study to the formulas in agriculture.
10. They encourage pupils to make use of aptitude tests available from the local, state, and college guidance services.

Although the activities just indicated are primarily ones of the individual teacher and his pupils, there are many other activities that may involve several teachers, guidance personnel, the administration, and certain lay persons. The following are illustrative of this type:

1. Mathematics clubs are organized and sponsored by the mathematics teachers.
2. Scientists, engineers, and business executives are invited to present the opportunities and the requirements for success in their area of specialization at assemblies and classes.
3. Assembly programs are conducted which show the need and importance of an understanding of mathematics for all educated persons.
4. Mathematics exhibits from schools of the city, county, or region are sponsored.
5. "Career days" are held each year in co-operation with local civic clubs.
6. Co-operation of psychiatrist and psychologist is obtained for special problems in guidance.
7. The mathematics teachers work with the science teachers in conducting a science and mathematics fair, an exhibit, and an open house. In some cases the specialists in the community guide the pupils in the planning of their projects and similar local specialists serve as judges of the exhibits. Valuable guidance is received by the pupil from these local businessmen, technicians, and craftsmen.
8. A survey of the occupational interests of the pupils and occupational needs of the community is conducted as a co-operative project of the guidance personnel and the mathematics department. The data may lend themselves to statistical treatment by the pupils in the mathematics classes. Thus in addition to obtaining valuable guidance information, it may make the study of mathematics more meaningful.
9. Mathematics teachers work with other school personnel in planning a "Choose-Your-College-Conference." Representatives of several colleges discuss with both pupils and parents the opportunities and requirements of their institutions.
10. Mathematics teachers with the guidance personnel and administration arrange field trips of small groups of pupils to professional or occupational activities in which the pupil plans to specialize.
11. Mathematics teachers work with guidance personnel in providing inventory and aptitude tests for pupils.
12. Mathematics teachers sponsor clinics to help the pupil succeed in the area of his specialization. For example, a clinic in how to read and study mathematics and scientific material; a clinic in review of basic mathematics understandings; a clinic in emotional behavior for success in mathematics and its related fields.
13. Mathematics teachers arrange for small groups of pupils to attend a

meeting of members of their chosen profession.

14. Mathematics teachers encourage and assist pupils in preparing skits and informational programs for television and radio presentations on the uses and importance of mathematics.

#### PERSONAL GUIDANCE

Many of the activities just mentioned furnish effective guidance to groups of pupils. However, the greatest single factor in influencing the pupil is the personal contact. It may be a brief remark in the hall about the behavior pattern necessary for success in a particular field or the quiet talk at the dinner table in the teacher's home about one's place in a civilization based on mathematics. It may be during a

few extra minutes when a teacher helped a pupil to understand the meaning of an exponent rather than pass him by as one who could not learn. It may be when the superior boy was given a challenging project to present to the class. In any case, as one picks out, in retrospect, those turning points or big influences in one's life, they seem to be centered in a few simple words or kindly relationships between two understanding individuals.

For the teacher to be prepared to make use of these opportunities for the pupil's profit requires a knowledge of the pupil and his environment plus tact, wisdom, patience, and labor far beyond the call of duty. The reward for success in guidance is also great. The remuneration can be only fully understood by the teacher who has helped pupils to successful and fruitful careers and occupations.

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#### UCLA Conference

The California Conference for Teachers of Mathematics is holding its fourth annual meeting on the Los Angeles Campus of the University of California during the period July 6-16, 1954. The Conference is sponsored by the University in co-operation with the California Mathematics Council. General sessions include a wide variety of lectures, panel discussions, and campus tours. The choice of study groups will satisfy a wide range of individual interests. Of special interest

are the laboratory groups in elementary and secondary mathematics where teachers may actually learn to make many of the teaching aids which are so necessary in our modern schools. Two units of credit may be earned by those participating in the Conference. A moderate fee is charged. For further information write to Clifford Bell, Mathematics Extension, University of California, Los Angeles 24, California.

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#### University of Wisconsin Conference

The sixth annual Conference on Teaching Mathematics will be held at the University of Wisconsin July 12-15. This year the program for the conference is planned in two parts. On July 12 and 13 the program will be concerned with the teaching of arithmetic, and July 14 and 15 with the teaching of secondary mathematics. Both for the Conference on Teaching Arithmetic and the Conference on Teaching Sec-

ondary Mathematics the program will include papers on research and recent investigations by national leaders in the field. Provision will be made for discussion of the papers presented.

The conference is sponsored by the School of Education of the University of Wisconsin and by the Wisconsin Mathematics Council. Requests for additional information should be addressed to Professor John R. Mayor, Education Building, Madison 6, Wisconsin.

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#### Louisiana State University's Institute

Louisiana State University's Fifth Annual Mathematics Institute will be held from June 20-25. Included on the program are a geometry laboratory and discussion groups in algebra, geometry, arithmetic, junior high-school mathematics, and enrichment materials led by experts in these areas. There will be lectures given by outstanding people in mathematics and related

fields. Excellent accommodations will be provided on the campus at reasonable rates. A vacation trip to interesting Louisiana can include this week for profit and enjoyment on the beautiful Louisiana State University campus. A copy of the program and additional information may be obtained by writing Houston T. Karnes, Director, Mathematics Institute, Louisiana State University, Baton Rouge, Louisiana.

# The mathematics required for graduation from high school<sup>1</sup>

W. I. LAYTON of *Stephen F. Austin State College, Nacogdoches, Texas*,  
*points out that the national mean of required units in English*  
*for the four-year high school is four times that of mathematics.*  
*In the three-year programs (grades ten, eleven, and twelve)*  
*this same comparison yields a factor of eleven.*

## INTRODUCTION

The purpose of this study is to try to determine the kind and amount of mathematics which each of the forty-eight states and the District of Columbia require for graduation from the high schools located within their boundaries. The data included are based upon material made available to the writer in 1952 by the state departments of education.

Of particular interest in connection with the present study is the report, "Requirements and High School Students' Programs."<sup>2</sup> The first part of this report "identifies the minimum requirements for graduation from secondary schools, reported by 48 States as of January 1, 1948."<sup>3</sup> While the present paper is devoted largely to the mathematical side of requirements for graduation from high school, the Office of Education study was made "to learn the extent to which the social studies are required or customarily taught in high schools and to locate some patterns of high-school subjects which meet students' personal educational wants

and also provide increased time for study in the social sciences."<sup>4</sup>

Forty-seven states and the District of Columbia are included in the present survey, and consequently it represents nation-wide practices. One state has not been analyzed due to insufficient data. Subsequently in this paper the District of Columbia will be counted as one of the states in order to permit more convenient wording.

Of the forty-eight states included in this study only four have no state-wide requirements in specific subjects for graduation from high school. In these four cases the requirements are left in the hands of local authorities. In some instances, however, where local groups work out the graduation requirements, these requirements are subject to review by some state authority. In the other forty-four states certain subjects are specified on a state-wide basis either by statute, by some organization with state-wide authority, or by a combination of these two. In the data which follow, zeros have been counted wherever there is no state-wide requirement in a certain subject for graduation from high school.

The unit is used as the basis for recording subject-matter requirements in this investigation. By unit we mean credit

<sup>1</sup> Based upon a paper presented at the Thirteenth Christmas Meeting of The National Council of Teachers of Mathematics at Lincoln, Nebraska, December 29-31, 1952.

<sup>2</sup> "Requirements and High School Students' Programs," Circular No. 300, Federal Security Agency, Office of Education, Washington, D.C., February, 1949. (Mimeographed.)

<sup>3</sup> *Ibid.*, Foreword by Galen Jones.

<sup>4</sup> *Ibid.*

in a class which meets a minimum of forty minutes a day for 172 days. In states where a system of record-keeping was used which did not employ units, the credits were translated in terms of units.

Provision is made for analysis of requirements for graduation from four-year high schools with grades nine through twelve or eight through eleven, and from three-year high schools with grades ten through twelve or nine through eleven. There are several combinations of grades found in high schools throughout the nation. However, for purposes of comparison, data were compiled only for the last four and/or the last three years of high school. The major emphasis in this study is on four-year programs since most of the states set forth their graduation requirements on a four-year basis.

While we are primarily interested in the mathematical requirements for graduation, in this paper we are considering for purposes of comparison units required in English, social studies, and science including laboratory science. We shall also be concerned with specific mathematics courses required for different curricula, and mathematics courses not required but recommended by state agencies for their high schools. In addition, the number of elective hours available to students in the three- and four-year programs will be investigated.

As has been mentioned before, there are several grade combinations in the high schools of the various states. Means have been computed for the last four years of the various high-school programs for certain subjects required of all students. In some cases the last four years consist of grades nine through twelve and in others of grades eight through eleven. We shall call these means which cover four years of high-school work four-year means.

#### GRADUATION REQUIREMENTS IN CERTAIN SUBJECTS

It is interesting to find that the four-year mean in the case of required mathe-

matics is .6 of a unit. This mean refers to mathematics required of all students in their last four years in high school. The requirements range from zero to two units with one unit being the modal requirement. Twenty of the forty-eight states included in the investigation require no mathematics. Twenty-four specify one unit. One state says one and one-half units, and one calls for two units. Thus twenty-six states, or 57 per cent of those which follow a four-year plan of organization in their high schools, require some mathematical training. The fact that twenty-six states require mathematics is some improvement over the situation existing in 1948 according to the study by the Office of Education.<sup>5</sup>

Turning now to the four-year mean for English in the present study we find that it is 2.8 units ranging from zero in eight states to four units in thirteen states. The modal requirement in English for these four-year programs is three units. Eighty-three per cent of the forty-eight states (including the District of Columbia) described in this investigation require some training in English in high school.

In the case of social studies the four-year mean is 1.7 units. The range is from zero to 3.5 units. The modal requirement is two units. Ninety-one per cent of the states require training in social studies. In one of these states the requirement in social studies is waived for those who have either voluntarily entered or been drafted into the armed forces. Another state specifies that an examination in American history and the United States Constitution must be passed for graduation from high school. This state has not been included in the above 91 per cent of the states which require training in social studies.

In science including laboratory science the four-year mean is .7 of a unit with requirements ranging from zero to two units. Twenty states require no training

<sup>5</sup> *Ibid.*, p. 6.

in science and nineteen require one unit. Six states require two units in science of all students during the last four years in high school. Fifty-seven per cent of the states under consideration call for some training in science during these four years.

Thus, as is evident from Table 1, the mean in English is about four and one-half times that in mathematics. The social studies mean is almost three times the mathematics mean, and science and mathematics are approximately the same although mathematics ranks at the foot of the list of subjects considered.

TABLE 1  
NATION-WIDE MEANS OF REQUIRED UNITS

Subject	Four-Year Programs	Three-Year Programs
English	2.8	2.3
Mathematics	.6	.2
Science	.7	.3
Social Studies	1.7	1.6

The preceding discussion has dealt entirely with four-year senior high-school programs. Nine of the states describe three-year courses of study. These may be grades ten through twelve or nine through eleven. As can be seen in Table 1, the mean of English required in these three-year programs is 2.3 units. The range is from zero to three units with three units being the most popular requirement.

In social studies the mean is 1.6 units, and the requirements range from one to two units with two units appearing most frequently.

The average requirement in science is .3 of a unit. One of the states which was recorded as zero in the computation of this average requires a unit in science in grades ten through twelve unless this work has been completed in the ninth grade of an approved junior high school. In the nine states following these three-year programs either no science is required or one unit is specified.

The mean of mathematics required in

the last three years is .2 of a unit. As was true in the case of science, one of the states which was called zero in the computing requires one unit in mathematics unless it has been completed in the ninth grade of an approved junior high school. These nine states either have no requirement in mathematics or they specify one unit in the last three years of high school.

The mean in English is at least eleven times the mean in mathematics, while the mean in social studies is about eight times that of mathematics. Science and mathematics hold approximately the same position. Referring to Table 1 again, we observe that mathematics seems to fare better in the four-year requirements as outlined by the states than in the three-year requirements. Probably this is true because where mathematics is required by a state for graduation from high school this requirement is usually attached to grade nine of a twelve-grade system, and the three-year program often does not include grade nine.

#### ELECTIVE UNITS

Let us examine now the elective units permitted by the states in the last four years of their high-school courses of study. Thirty-three of the states included in this survey have provisions concerning electives which could be clearly evaluated. The mean of the minimum number of elective units permitted in these four-year programs is 9.2. This is approximately two and one-third years of high-school work, or about 58 per cent of the time the student has to spend during the four years under consideration. The range of the minimum number of elective units permitted by these states is from five and one-half to sixteen units.

As we have already indicated, 91 per cent of the states specify training in social studies during the last four years of high school, and 83 per cent in English. Also, 57 per cent of the states require mathematics in the four-year programs and the same number require science.

As was previously pointed out, twenty of the forty-eight states require no mathematics in the last four years of high school. Among these twenty states elective units range from seven and one-half to sixteen. It would seem, therefore, that these states could require some mathematics if they deemed it advisable.

In the case of the five states whose policies concerning electives in their last three years of high school could be clearly followed, we find that the mean of these elective units is approximately seven. The number of elective hours varies from a minimum of five to a maximum of eight. Apparently there is time for some mathematics to be included during these last three years in high school.

#### SPECIFIC MATHEMATICS COURSES REQUIRED

We shall now consider specific mathematics courses which are required by the states of all students in their last four years of high-school work. Ten of the states specify that certain mathematics courses be taken. All but one of these states stipulate one unit, and the remaining state requires one and one-half units. Three of the states call for general mathematics or first year algebra. In the other states the courses from which the required unit in mathematics may be chosen are general mathematics, algebra, arithmetic, and business arithmetic. One state attempts to insure an adequate knowledge of arithmetic for its prospective graduates by requiring that one-half unit in senior arithmetic be included in the one and one-half units in mathematics required of all students. If senior arithmetic is not included, two units must be taken instead of one and one-half.

#### MATHEMATICS COURSES RECOMMENDED

Ten states make recommendations concerning the mathematics to be taken by students during their last four years of high school. Some recommend one or more units in mathematics while others

suggest certain courses as being desirable such as algebra, general mathematics, applied mathematics, arithmetic, and commercial arithmetic. One state, while not making a definite recommendation concerning mathematics, is interested in whether or not each school has a plan for conserving and improving the fundamental skills of its pupils in arithmetic, spelling, reading, English usage, and ability to think as evidenced by clear expression.

#### MATHEMATICS FOR SPECIALIZED CURRICULA

We have already considered the mathematics required by the states for all of their students in the last four years of their high-school programs. Now let us examine the mathematics for specialized curricula during these last four years. In one state plane geometry one unit, solid geometry one-half unit, and advanced algebra one-half unit are required in the academic curriculum. One unit in plane geometry is listed in the scientific curriculum.

In another state those enrolled in vocational or nonacademic curricula may take a unit in applied mathematics as the year of required mathematics. By applied mathematics is meant any mathematics course especially organized to meet the needs of pupils taking prevocational or vocational shop, business, home economics, or other approved specialized courses.

The following practices are each used by one state:

One unit in mathematics must be taken by academic students in addition to the unit required of all students.

One unit in business arithmetic is specified for vocational bookkeeping, vocational office practice, co-operative retailing, and nonvocational business.

Two units in mathematics are stipulated in the college preparatory curricula.

One of the states requires that the candidate for graduation from high school attain a grade of 75 per cent or better in

each of the units that are listed for the academic or the scientific diploma.

#### SUMMARY AND CONCLUSIONS

Let us now examine some of the more significant findings of the investigation of state requirements for graduation from high school in forty-seven states and the District of Columbia. In the four-year programs when compared with English, social studies, and science, mathematics ranks at the bottom of the list of means being preceded rather closely by science. It is evident from the data that much emphasis is being given by the states to two of the subjects included in this study, namely, English and social studies.

Percentages tell much the same story. In these four-year programs mathematics and science are required by a much smaller percentage of the states than are English and social studies.

In those few states which describe three-year courses of study the picture is even more bleak, mathematically speaking. Mathematics and science as compared with English and social studies occupy a less favorable position in the three-year curricula than in the four-year.

The material we have presented dealing with electives also seems of some importance. With the amount of time apparently

available in the courses of study in both the three-year and the four-year programs it seems that more mathematics could be included for graduation. Also, the addition of the twelfth grade in schools that have heretofore been operating on the eleven-grade plan offers an opportunity to include more mathematics.

Some of the states specify that in the four-year programs certain mathematics courses must be taken for graduation from high school, while others merely set forth recommendations. However, recommendations are not necessarily followed, and we have been concentrating largely on requirements rather than recommendations.

A few states require training in mathematics for specialized four-year curricula such as academic, scientific, or vocational.

Perhaps some committee of the National Council of Teachers of Mathematics may be interested in working out recommendations concerning the kind and amount of mathematics which should be required for graduation from high school. It is to be hoped that it will be deemed advisable to increase the amount of required mathematics as we train our young people to take their places in the highly technical and scientific age in which they live.

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### Have you read?

FITZSIMONS, FRANK P. "The Case Method of Teaching," *School and Society*, October 3, 1953, pp. 102-7.

Harvard University introduced the "case method" back in 1869, under much protest, into its Law School. The author maintains this bilateral method is more conducive to developing student responsibility because the student must think for himself. But this is not all; the instructor is also constantly in a changing situation. The "case method" consists of posing a problem which is followed by give-and-take discussion, with critical class analysis and the ultimate goal a compilation of the principle or solution resolved. This is a *whole* method with many of the facets of the case considered as well as many by-products of learning which remain as residue. This is not a mathematical article

but you will want to read it for its implications in the teaching of mathematics.

WEAVER, J. FRED. "Differentiated Instruction in Arithmetic—An Overview and a Promising Trend," *Education*, January 1954, pp. 300-305.

This promising trend as viewed by the author deals with levels of learning in action. Students given freedom to use their own methods will attack problems in new situations with methods at their command. This means that differentiation may come through approach as well as content. This provides another valuable approach in our search for better ways of providing for all the students all the time.—Philip Peak, *Indiana University, Bloomington, Indiana.*

# Vector representation of multiplication and division of complex numbers

JUAN E. SORNITO, *Central Philippine College, Iloilo City, Philippines,*  
presents a simple and interesting way to represent  
the multiplication and division of complex numbers  
by means of vector representations.

ONE QUESTION raised by students in mathematics is how to represent multiplication and division of complex numbers by vectors in rectangular co-ordinates. The purpose of this brief paper is to present a simple explanation that can be given to students.

## ADDITION AND SUBTRACTION

The vector representations of addition and subtraction of complex numbers are familiar to students of mathematics. The examples presented below are intended to serve as a basis for the explanation of addition and subtraction.

EXAMPLE:

$$(3+4i) + (2-3i) = (5+i).$$

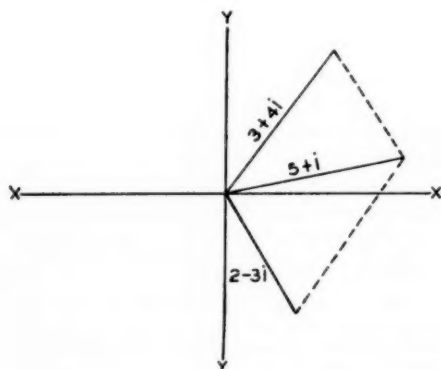


Figure 1

Figure 1 illustrates the process of addition. It is accomplished by plotting the

quantities to be added and completing the parallelogram of which the vectors are sides. The diagonal drawn from the origin is the vector sum of the quantities. Subtraction is accomplished in the same manner, except that the vector representing the subtrahend is reversed before addition is performed. This reversal of vector is the geometric interpretation of the algebraic principle of changing the signs of the subtrahend.

EXAMPLE:

$$(7+3i) - (3-i) = (7+3i) + (-3+i) \\ = (4+4i).$$

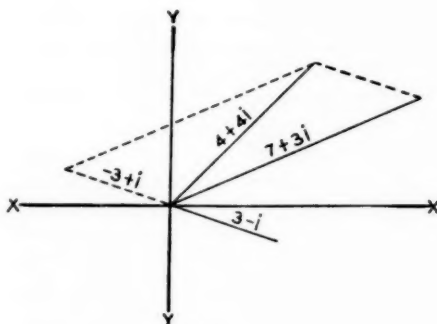


Figure 2

It will be noticed in Figure 2 that the subtrahend  $(3-i)$  is equal in magnitude but opposite in direction to the vector of  $(-3+i)$ . The rest of the operation is the same as in addition.

## MULTIPLICATION

A simple explanation for the vector representation of multiplication of two complex numbers can be given if use is made of an elementary concept of multiplication. In the arithmetic of whole numbers multiplication is defined as the result of taking the multiplicand as many times as indicated by the multiplier. Thus  $3 \times 5$  equals or is the same as  $5+5+5$  or 15.

### EXAMPLE:

(a)  $(2+i)(3+2i)$

(b)  $(2-3i)(3+2i)$

### Algebraic representation:

(a)  $2(3+2i) + i(3+2i)$

$$6+4i+3i+2i^2$$

$$6+7i-2$$

$$4+7i$$

(b)  $2(3+2i) - 3i(3+2i)$

$$6+4i-9i-6i^2$$

$$6-5i+6$$

$$12-5i$$

### Vector representation:

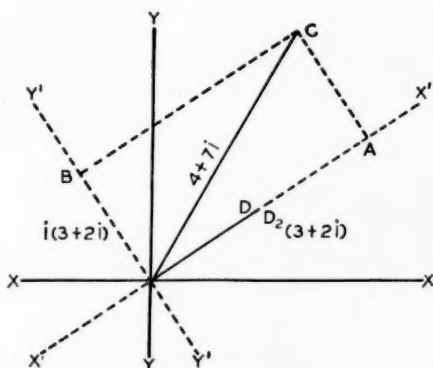


Figure 3(a)

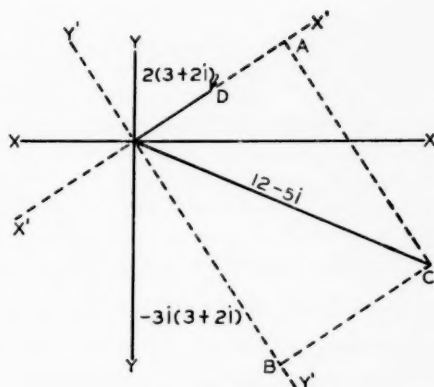


Figure 3(b)

**PROCEDURE:** Plot the multiplicand  $3+2i$  on the horizontal and vertical axes as in Figure 3(a). Draw new axes  $X'Y'$  and  $Y'Y'$  along the vector  $3+2i$  and perpendicular to this vector. On the new axes measure the magnitude of the multiplicand ( $3+2i$ ) or the vector  $OD$ , two times as indicated by the real component of the multiplier ( $2+i$ ), along the positive direction of the new axis of reals. Along the positive direction of the new axis of imaginaries, measure the vector  $OD$  once as indicated by the imaginary component of the multiplier. The sum of the vectors  $OA$  and  $OB$  is the vector product of the complex quantities. In Figure 3(b) it will be observed that the vector magnitude of the multiplicand  $3+2i$  is laid off three times along the negative direction of the axis of imaginaries.

The addition of the vectors  $OA$  and  $OB$  is the geometric equivalent of the algebraic expressions  $2(3+2i)$  plus  $i(3+2i)$  in Figure 3(a) or  $2(3+2i)$  plus  $-3i(3+2i)$  in Figure 3(b). Hence the vector sum  $OC$  in each case is the vector product of  $(3+2i)(2+i)$  in Figure 3(a) and  $(3+2i)(2-3i)$  in Figure 3(b) respectively. The multiplier is not plotted because it is an abstract quantity indicating only how many times the multiplicand is to be taken.

## DIVISION

Division is just the reverse process of multiplication. Remember that the arith-

metrical concept of division is that it is a process of continued subtraction to determine how many times the divisor should be subtracted from the dividend until there is nothing left in the latter. Thus,  $20 \div 5$  is equivalent to  $20 - 5 - 5 - 5 - 5 = 0$ . This means that after subtracting 5 from 20 four times, there is nothing left; hence the quotient is 4. In complex quantities the operation is performed as follows:

EXAMPLE: Solve  $(18+i) \div (4+3i)$

Algebraic solution:

$$\begin{aligned}\frac{(18+i)}{(4+3i)} &= \frac{(18+i)(4-3i)}{(4+3i)(4-3i)} = \frac{(72-50i-3i^2)}{(16-9i^2)} \\ \frac{(72-50i+3)}{(16+9)} &= \frac{(75-50i)}{25} = (3-2i).\end{aligned}$$

Solution by vectors:

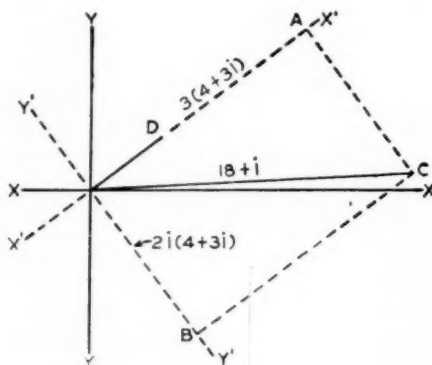


Figure 4

PROCEDURE: Plot the dividend and the divisor on the horizontal and vertical axes as shown in Figure 4. Draw a new set of axes so that the new axis of reals coincides with the vector of the divisor. The new axes would now be  $X'X''$  and  $Y'Y''$  representing the axis of reals and the axis of imaginaries, respectively. From the end  $C$  ( $18, 1$ ) of the vector of the dividend drop perpendiculars to the new axes intersecting them at  $A$  and  $B$  respectively. The

(Continued on page 382)

## The editor's mail

The January issue of *THE MATHEMATICS TEACHER* arrived either Monday or Tuesday, I don't recall which, and I read the entire issue at one sitting that evening. I intended to get a letter off to you that very day but too much other correspondence pushed it aside. I certainly congratulate you on the "new look." The change in format has done much to liven up the looks of the journal. I also like the arrangement of printing matter in the new headings that you have created. It has a very refreshing look.

As to the actual content I think it is excellent. I was particularly interested in the articles by Shuster and Greunberger. There are at least two reasons why I like those articles. First, they are short and to the point. Second, they can be used in classroom discussion. For example I have a beginning freshman class that happened to be talking about imaginary numbers the very day that the journal arrived. The next day I gave the journal to one of the boys who was interested in electricity, and asked him to read it and report to the class. He seemed to enjoy it and did an excellent job. I think all teachers appreciate articles of that kind. I am also glad to see the complete list of officers of the Council.

(signed)

H. G. AYRE

Western Illinois State College

My attention has been called to the article by Merton Taylor Goodrich, "A Systematic Method of Finding Pythagorean Numbers," which appeared in the *National Mathematics Magazine*, Vol. XIX, No. 8, May, 1945. In this paper Professor Goodrich derives the equivalent of the equations (5) as given in my article, "Pythagorean Numbers" of the January issue of *THE MATHEMATICS TEACHER*, and he also presents a method for tabulating Pythagorean numbers which is somewhat different from the method I proposed.

I regret that I did not see this paper by Professor Goodrich in time to make appropriate reference to it in my article; however, I would like to take this opportunity now to make acknowledgment of it.

(signed)

PHILIP J. HART

Associate Professor of Physics  
Utah State Agricultural College  
Logan, Utah

Congratulations on the superbly re-designed *MATHEMATICS TEACHER*.

Through the thirty-three years of its existence it has been an indispensable companion of teachers of mathematics, and this present enhancement of its aesthetic aspects will add much to its stimulating helpfulness.

(signed)

OSCAR J. PETERSON, Head  
Department of Mathematics  
Kansas State Teachers College  
Emporia, Kansas

# A comparative study of the effectiveness of lessons on the slide rule presented via television and in person<sup>1</sup>

GEORGE R. ANDERSON, *Millersville State Teachers College, Millersville, Pennsylvania*, and ABRAM W. VANDER MEER, *Pennsylvania State College, State College, Pennsylvania*, find that television "probably could be especially helpful when the supply of first-rate instructors is limited" and conclude that television is not likely to replace the present-day classroom.

## INTRODUCTION

Much has been said and written concerning the educational importance of television. Perhaps greater interest has been shown in the general cultural impact of this new communications medium, but its potentialities for direct instruction have not been ignored. Public school systems, universities, and the armed services all have made some efforts in the direction of formalized instruction via television. In general, these efforts appear to have met with some success.

Up to the present time, the degree to which students learn from television lessons as compared with the same lessons presented in person has not been fully studied by controlled experimental meth-

ods. The purpose of the study reported here was to determine the relative effectiveness with which television could be used to present a unit of work on the slide rule in high-school mathematics classrooms.

The senior author who was responsible for the operational conduct of the experiment has been moderator and college producer of a series of general informational television programs for the past four years. Included in this work has been a series of three programs on the slide rule.

## PROCEDURE

Five classes of high-school sophomores ranging in number from twenty-one to twenty-six were involved in the experiment. The content of the experimental instruction concerned certain computational skills on the slide rule and was organized in six half-hour programs for a six weeks period. A five-item test on the lesson was given at the end of each instruction period. Individual programs covered the following:

1. Background materials, reading the D scale, elements of multiplication,
2. Review of reading the D scale, finding reciprocals, division, combined multiplication and division,

<sup>1</sup> The following individuals and firms rendered assistance and service without which this experiment would have been impossible and for which sincere gratitude is expressed: Mr. Paul Rodenhauer and staff of WGAL-TV who put the station on the air one-half hour earlier so that class time schedules might be met; Mr. Harry Seelen of the Radio Corporation of America through whose efforts two 21-inch television sets were loaned; the Keuffel and Esser Company which loaned one hundred beginners' slide rules; the co-operating teachers in the various high schools. Especially helpful in the planning were Dr. Lee E. Boyer and Dr. Paul Rummel of the Millersville State Teachers College faculty; and Dr. Hugh Davison, Professor of Research and Statistics, Pennsylvania State College.

3. Reading A and B scales, squares and square roots, areas of circles,
4. Reading K scale, cubes, cube roots, elements of proportion,
5. Proportion and applications thereof,
6. Review.

The groups taught by television met in their regular classrooms and viewed the presentation from 9:45 to 10:15 A.M. each Wednesday morning for six consecutive weeks. The lessons taught "in person" were presented on the same day in order to prevent over-night and after-school exchange of test items and to help insure uniformity in lesson presentation.

In the course of the experiment, one modification of the foregoing procedure was deemed necessary. Scores on a test administered after teaching the first lesson were so low that it seemed advisable to reteach this lesson completely. This unfortunately resulted in the omission of the review planned at the end of the course because of the limited availability of television broadcast time.

The final examination was made by combining all six of the tests which were given at the end of each instruction period.

#### EXPERIMENTAL POPULATIONS

Three classes of tenth-grade mathematics students were taught via television by the senior author. The largest of these groups contained twenty-six students. The experimental television broadcasts were

viewed on 21-inch receivers placed in the regular classrooms.

Two other classes of the same size were taught in person by the senior author. All were in the service area of WGAL-TV in Lancaster, Pennsylvania.

Before instruction began, the California Test of Mental Maturity, Advanced Series, and the Stanford Achievement Test in Advanced Arithmetic, Form E, were administered. On the basis of these tests two matched groups of forty-one students each were chosen.

The over-all equivalence of the groups is shown in the first two columns of Table 1. The "in-person" group was composed of eighteen girls and twenty-three boys while the television group had in it seventeen girls and twenty-four boys. The fact that the latter group contained but one more boy than the former could scarcely contribute any significant difference due to sex. Students were not aware that they would or would not be included in these matched groups.

The equivalence of the groups is shown in Table 1. It will be noted that although the in-person group had the smaller percentage of girls, it also had slightly higher mean intelligence and arithmetic scores.

#### CRITERION TESTS

A five question test on each weekly lesson was devised and administered at the end of each period of instruction. The

TABLE 1  
CHARACTERISTICS OF EXPERIMENTAL GROUPS

	No.	Percent- age of Males	Percent- age of Females	Intelligence Quotients (Means and Sigmas)			Arithmetic Computation (Means and Sigmas)		
				Total	Male	Female	Total	Male	Female
Television Group	41	59	41	110.70	109.46 10.84	112.47 10.87	75.17 9.72	75.71 10.64	74.41 8.93
In-Person Group	41	56	44	112.05	110.61 10.88	113.89 11.38	75.15 9.72	76.39 9.43	73.56 9.84

twenty-five questions comprising the five short tests were re-administered as the final examination on the day after the last instruction was given. It did not seem feasible or desirable to make these tests completely objective. However, the following procedure was followed with a view to eliminating chance and capricious errors in scoring: (1) for single step operations a difference of plus or minus one in the third place was counted as correct, (2) for two-step operations (where the rule must be adjusted twice) plus or minus two in the third place was allowed, (3) in any case where the answer was obviously obtained by arithmetic instead of using the rule, the answer was marked wrong. For example, an acceptable answer for  $64.3 \times 71.5$  might be 4600 for slide rule. The exact product is 4597.45 but the student could hardly have read this accuracy on the rule and hence must have done it by arithmetic, (4) when the sequence of digits was correct, but the decimal point incorrectly placed, half credit for the problem was given. It was reasoned that the object of the experiment was to learn how well one could learn to use the slide rule and the correct sequence of digits indicated that the rule was manipulated properly.

## RESULTS

The over-all effectiveness of television in teaching the experimental groups to use the slide rule is shown in Table 2. While the mean score on the final test made by the group taught "in person" was approximately 3.5 percentage points higher than that of the group taught by television, this difference was not statistically significant. It would seem, therefore, teaching the slide rule via television is practically as effective as teaching it in person.

The data were analyzed to determine whether differences would be found when the experimental groups were divided according to sex and according to intelligence test scores. The results of these analyses are also summarized in Table 2. Again, no significant differences were found. Table 2 also shows that the males contributed more to the superiority of the "in-person" group than did the females. The original assumption that boys would learn the slide rule more readily than girls tends to be supported by the data in Table 2, although none of the differences favoring the boys was statistically significant.

None of the observed groups or subgroups scored as high as 50 per cent, on

TABLE 2  
MEANS AND STANDARD DEVIATIONS ON TESTS ON THE SLIDE RULE

	Sum of Weekly Tests		Final Tests				
			Male	Female	High I.Q. <sup>1</sup>	Low I.Q. <sup>2</sup>	Total
Television	mean	43.34	39.83	35.29	44.5	31.1	37.95
	$\sigma$	19.61	18.92	15.17	18.3	13.8	17.91
In Person	mean	42.41	45.17 <sup>3</sup>	36.67 <sup>3</sup>	47.8	32.4	41.44
	$\sigma$	19.23	16.29	18.91	19.1	11.3	17.99
Differences (In Person Minus Television)		-3.83	5.34	1.38	3.2	1.3	3.49
Critical Ratios		1.26	1.04	0.24	0.43	0.59	1.61

<sup>1</sup> High I.Q. is defined for purposes of this study as 110 and above; low I.Q. as 109 and lower.

<sup>2</sup> This is the largest observed difference between males and females, and has a critical ratio of 1.52.

the average, on the final examination. This indicates an unsatisfactory degree of learning for all groups.

As measured by the difference between the sum of the weekly lesson test scores and the final test scores, it appears that the groups taught by television forgot what they had learned more readily than groups taught in person. The difference between mean weekly test score and final test score was highly significant (C. R. 3.25) for the television group, but not for the in-person group (C. R. 0.88). The correlation between these scores was .61 for the television group and .81 for the in-person group.

#### OBSERVATIONS

One of the greatest difficulties in teaching via television is pacing one's presentation. In the classroom, the experienced teacher can tell much about how well his material is being understood by attending to such things as facial expressions, lack of interest, restlessness, and number of questions. None of these is observable by the television teacher. A representative studio class might remedy this situation just as a studio audience aids an actor to time and gauge his presentation. It would also aid in overcoming another big drawback—lack of opportunity for the learners to ask questions. If the group were of average ability or even less, they would likely ask representative questions such as might occur to those viewing the program. This might supply a bit of repetition where needed most and when needed most.

A number of general observations were made by co-operating teachers while the study was underway. Among these were: (1) attention was as good for the classes viewing television as it was for ordinary teacher presentations. Those disinterested in slide rule were equally disinterested in the tenth-grade mathematics they were then studying, (2) co-operating teachers and students alike generally agreed that the poor showing on the final examination

was due to lack of practice rather than lack of understanding. A few students took the attitude that this work was somewhat extracurricular in nature and that they did not need to study with the same sincerity as they did their regular class-work, (3) some students would have preferred to be in a group other than the one in which they found themselves; opinion was about equally divided here, (4) the television groups were unanimous in stating that they would liked to have had the opportunity to ask questions. One student said: "When we got lost on one point we missed most of the rest of the presentation," (5) television viewers said it was difficult to look at their rules and at the same time look at the screen. It is difficult for the writer, however, to see much difference between viewing an eight-foot demonstration rule such as that used in "in-person" lessons and the 21-inch television screen used by television groups. Probably the point is more closely related to that of timing the presentation than difficulties of viewing. It is very likely that those viewing the screen saw the rule more distinctly than did those in the "in-person" group since the portion as shown on the 21-inch receivers magnified that section of the rule itself. In addition the television receiver showed only that section of the rule under study, thereby facilitating the finding of the desired numbers.

#### CONCLUSIONS

In times of national emergency or whenever large numbers of people must be taught skill subjects, television with a studio class and auxiliary instructors for drill purposes might do the job effectively. This medium probably could be especially helpful when the supply of first-rate instructors is limited.

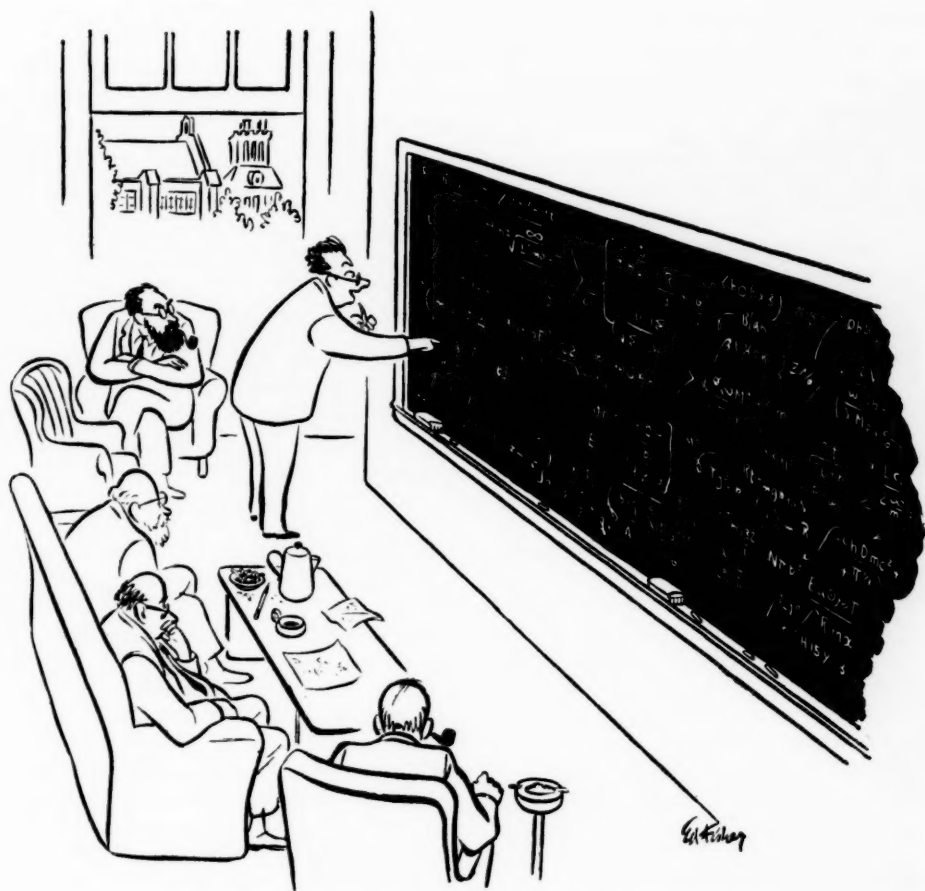
Television is not likely to replace regular teaching. The timing problem is even more difficult with television than with motion pictures, for example. The television program occurs once and is gone

while a film can be shown repeatedly. Television is an expensive medium. The minimum cost of the six one-half hour programs on this relatively small station would be \$3,000 if they had to be paid for at usual rates. Few public schools have sufficient funds to carry on such programs regularly. Commercial stations have been generous in their donation of time for educational purposes and may continue to do

so if they feel that it is in the public interest.

Television classes do not have the advantage of inter-class discussion under the direction of the instructor. It may be, therefore, that television would be most effective for presentation of factual materials.

Further research is greatly needed to shed light on the foregoing problems.



"Say, I think I see where we went off. Isn't eight times seven fifty-six?"

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## • AIDS TO TEACHING

*Edited by Henry W. Syer, Boston University Boston, Massachusetts, and  
Donovan A. Johnson, University of Minnesota, Minneapolis, Minnesota*

### BOOKLETS

#### *B.182—Engineering a Creative Profession*

Engineers Council for Professional Development, 29 West 39th Street, New York 18, N.Y.

Booklet; 6"×9"; 32 pages; \$0.25.

*Description:* The purpose of this booklet is to describe the engineering profession to the student who is considering engineering as a possible professional choice. It discusses topics such as the fields of engineering, the educational requirements, and the opportunities in engineering. Courses, aptitudes, and interests that are essential for success in engineering are specified. Each of the major branches of engineering such as civil, mining and metaphysical, chemical, electrical, and mechanical are described in terms of the kind of activities they involve.

*Appraisal:* The illustrations in blue, white, and black make this pamphlet attractive and add interest to the script. It is well written in a simple, direct style that should appeal to secondary pupils. Its emphasis on mathematics and science and the ability to think independently will be welcomed by mathematics teachers. Every high-school counselor should be supplied with a copy.

#### *B.183—Guidance Manual*

Engineers Council for Professional Development, 29 West 39th Street, New York 18, N.Y.

Booklet; 6"×9"; 16 pages; \$0.20.

*Description:* This manual is written to assist the high-school counselor or engineer in counseling young people about the engi-

neering profession. A questionnaire to be filled out by the student entitled "Shall I Study Engineering" is a separate appendix to the Guidance Manual. This questionnaire obtains information regarding the education, school activities, and work experience that can be helpful in predicting a student's likely success in engineering. The manual gives specific suggestions for conducting the interview and for topics to cover in an address on engineering.

*Appraisal:* This publication contains no information about engineering, its requirements or opportunities. Its emphasis is on the organization of local engineering groups to establish guidance or advisory committees. The treatment of how to counsel young men interested in engineering is very brief and sketchy.

#### *B.184—Math at General Electric*

Department 2-119 Public Relations, General Electric Company, 1 River Road, Schenectady 5, New York

Booklet; 8½"×11"; 15 pages; Free.

*Description:* This booklet begins by pointing out the importance of mathematics in our society as well as at General Electric. The balance of the publication illustrates twenty-two typical problems solved by General Electric people. The problems are completely different from each other and deal with topics such as computing power factors, earnings, a tax payment, spaces, and the location of a point. The problems illustrate the work of a meteorologist, an editor, an advertising manager, a carpenter, a machinist, an electrician, a glassblower, a toolmaker, a welder, an engineer, an accountant, a salesman, a laborer, and a stenographer.

Each problem describes briefly the setting and the complete solution.

*Appraisal:* The purpose of this publication is to stimulate a greater interest in mathematics so that ultimately there will be a greater supply of technically trained recruits for the ranks of modern industry. It should also stimulate a greater understanding of the applications of mathematics so that high-school students will have a better groundwork for living a richer and more useful life. The emphasis on the importance of mathematics and the wide variety of application written in a simple concise style will be a welcome addition to every mathematics classroom.

*B.185—Mathematics in Public High Schools*

Office of Education Bulletin, 1953, No. 5, Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C.

Booklet; 6"×9"; 47 pages; \$0.20.

*Description:* This new publication contains information on enrollments in mathematics and administrative provision for instruction in mathematics. The data include enrollments, offerings, size of classes, number of teachers, field trips, length of class periods, and other pertinent information about mathematics at each grade level—grades seven through twelve. In addition to brief descriptions of trends in mathematics education, sixty tables present the data in convenient form for the busy reader.

*Evaluation:* In view of the current criticism of education and the increased need for technical training it is essential that mathematics teachers, administration, counselors, and research workers have the answers to these questions. Are as many secondary students enrolled in mathematics now as there were ten years ago? What mathematics, if any, is required in secondary schools? What is the status of general mathematics? How large are mathematics classes? How frequently do mathematics teachers use field trips? This

publication provides the answers to these questions on the basis of information obtained from a sample of 857 public high schools.

*B.186—Telecomputing*

Telecomputing Corporation, 133 East Santa Anita Avenue, Burbank, California

Bulletin; 8½"×11"; 8 pages; Free.

*Description:* These bulletins are descriptions of automatic data-analyzing instruments produced by this manufacturer. These instruments are devices for automatic graphing of numerical data. The operation of the machines is discussed as well as the kinds of problems it can handle. In addition to complete specifications certain important parts are described in detail.

*Appraisal:* Telecomputers are expensive machines to be used by industry to build efficiency in analyzing data. These bulletins merely call to the teacher's attention an interesting sidelight of the problem of graphing data.

*B.187—Your Mathematics Notebook, Vol. 5, No. 1*

Scott, Foresman and Company, 433 E. Erie St., Chicago 11, Illinois

Booklet; 7"×9"; 4 pages; Free.

*Description:* This issue of the Scott, Foresman Service Bulletin has three articles of interest to every mathematics teacher. One article describes a Mathematics Day at Glenville, West Virginia. This project included student demonstrations, an exhibit, and a tea for visitors from other high schools. The second article describes changes in mathematics courses and current enrollments. The final article summarizes briefly the chapter on motivation in the twenty-first yearbook.

*Appraisal:* Although a trade publication with some advertising and a very brief treatment of topics, mathematics teachers should request to have their names placed on the mailing list.

## EQUIPMENT

### *E. 144—Alidade*

Elwood M. Stoddard, 47A Jones Street, Hingham, Massachusetts.

Kit; wood; 12" base; \$1.75.

*Description:* This kit consists of a 12-inch base, two 12-inch ruler strips, an 8-inch base block, an 8-inch sighting bar, a protractor, mirror, blocks, and all necessary nails, screws, bolts. All parts are of wood or masonite and are pre-cut and finished ready for assembly. The kit can easily be assembled with simple tools. Complete step-by-step instructions for assembly are included. This makes it possible for a novice to assemble the alidade in a very short time.

*Appraisal:* An alidade is an essential piece of equipment for every mathematics classroom. It is useful for field work on mapping, scale drawing, and indirect measurement. This inexpensive kit will give the pupils or teacher the pleasure of building a device that will give accurate results. The device is well designed and the pieces are accurately cut to make an excellent field instrument.

### *E. 145—Applied Calculus*

### *E. 146—Differential Calculus*

### *E. 147—Fundamental Identities*

### *E. 148—Integral Calculus*

Pocock Laboratories, 810 Sunset Lane, East Lansing, Michigan.

Game: 52 cards in each deck; playing card size; \$1.25 per deck.

*Description:* These games, called AQ (Answer the Question), are designed to help the student memorize important information. In general, half of the cards are marked Q, the other half A. Cards marked Q ask questions, such as differentials, integrals, or identities. Cards marked A provide the answers to these questions. The playing procedure for matching cards is very similar to Solitaire. Instead of building a consecutive sequence according to

suit, AQ is played by forming pairs of A and Q cards. If a player is uncertain of the correctness of his AQ pair, he can check by means of simple arithmetic addition problems given on the backs of the card. Equal sums indicate that the pair is a correct combination. The game is usually played by one person but two decks may be combined so that two players may "play double."

*Appraisal:* These cards are essentially drill cards designed to help commit to memory important formulas of calculus and fundamental identities. Elementary and secondary mathematics teachers have frequently found that games add interest and effectiveness to drill. Similarly, college mathematics teachers may find it of value to recommend painless drill to meet the competition for study time. The cards are of good quality material and have appropriate mathematical designs on the back side. The mechanics of playing can be quickly mastered.

### *E. 149—Triangles and Their Angles (T 1)*

### *E. 150—Quadrilaterals and Parallelograms and Their Diagonals (T 2)*

### *E. 151—Polygons (T 3)*

### *E. 152—Pythagorean Theorem (T 4)*

### *E. 153—Altitudes of Triangles (T 5)*

### *E. 154—Circle Circumscribed about a Triangle (T 6)*

### *E. 155—Triangles Equal in Area (T 7)*

### *E. 156—Angles Inscribed in a Semi-circle (T 8)*

### *E. 157—Medians of Triangles (T 9)*

### *E. 158—Perpendicular Bisector (T 10)*

Ideal School Supply Company, 8312-46 Birkhoff Avenue, Chicago 20, Illinois.

Demonstration boards; each 18"×24"; \$25.00 for set, \$3.00 each. (Order by using #855, followed by symbol in parentheses above.)

*Description:* Each board is made of sturdy masonite with rounded corners and

an attractively colored surface appropriate to the geometry concerned. All, except the "Pythagorean Theorem," have holes to hold pegs; the pegs are connected by black elastic thread which forms the outline of the geometric figures. These boards are used to demonstrate the concepts to the class, and their unique feature is the ability to show, *dynamically*, the figures growing, changing, and modifying their shapes and sizes. By studying the relationships which remain constant in the midst of all this change, important geometric facts are derived *inductively* and the important idea of *function* is introduced into geometry. The announcement says that there will be "Pupil Discovery Boards" to correspond to each large board, only 9"×12", and selling for \$5.00 for the set of ten, or \$0.60 each. (Order by catalog number 856, followed by P 1, etc.) In addition, "Pupil Work Sheets" (pads sufficient for 40 pupils, \$0.25, Catalog number 857 P I, etc.) are to be supplied, but the reviewer has not seen the pupil boards or pads.

*Appraisal:* Hurrah! At last the Burns Boards are available commercially. The tremendous advantage that we lose is the fun and learning involved when these are made by pupils or by teachers, but this is still not impossible, and some people may prefer the larger, sturdier models which now can be bought. They are very attractively designed and intelligently produced.

The first impression of anyone who has not used or seen these boards used is one of disappointment. It cannot be stressed too often that aids such as this are only *aids*; it still takes the thought, experience, and planning of the teacher to bring them to life. If you feel disappointed in trying them, don't give up too soon—essentially they are important and educationally sound devices which will add interest and understanding to the learning of geometry. A teacher who does not like them either does not understand how to use them, or teaches by a method (possibly a good one)

which does not need these devices; but it is more apt to be the former reason. The booklet (seven pages) of explanation which is provided helps a great deal, even though, as a booklet, it is not attractive nor anywhere complete enough: it needs pictures, copies of the pupils' work-sheets, and (for advertising purposes) the name and address of the producer. By using the *Eighteenth Yearbook* of the National Council of Teachers of Mathematics in addition, a fair idea can be derived of the teaching methods to be used with these Burns Boards. The best way, of course, is to see some vital teacher, like Frances Burns herself, use the devices and see what an integral part of the method they become.

In using these with a class the reviewer found a few adverse comments. Some said the boards with green backgrounds were hard to see at a distance; black thread against yellow was much more satisfactory. Others felt that all boards were excellent except the one on the Pythagorean theorem, and that *that* one was only a picture which could better be drawn on the blackboard, a lantern slide, or a piece of cardboard provided by the teacher. Unfortunately school budgets demand a close scrutiny of our purchases. In general, the boards are well worth the price and should be considered for purchase.

We discovered in using them that the pegs fitted rather tightly at first but soon seemed to loosen up. They did, however, mark the background slightly and after considerable use might reduce the attractiveness of the board. The hole for the thread was not in the center of each peg, and this seems sensible, but it did require an attention to which end of the peg you used; is this important, or should it be avoided by centering the hole? The position of the knots in the threads sometimes delayed the operation of the board, but experience will certainly teach one to avoid this trouble. Rather than end on these pessimistic notes we should say: these boards are well-made, attractive,

and connected with a valid teaching method; give them a try.

*E. 159—Craft Beads*

Waleo Bead Company, 37 West 37th Street, New York 18, N.Y.

Beads; Tile or Wood; Varied Sizes or Color.

*Description:* These beads may be obtained in a variety of shapes such as spherical, ellipsoid, cylindrical, and square. Sizes vary from  $1/16$  inch to  $3/8$  inch. Each size is available in a variety of brilliant colors.

*Appraisal:* Here is a supply of attractive beads that can be used for making counting devices such as an abacus to illustrate the meaning of numbers. At advanced levels they can be used for sampling experiments, number series, or probability tests.

*E. 160—Elementary Computer*

M. R. Thompson, Route 2, Box 19, Monmouth, Oregon

Slide Rule; Cardboard;  $5\frac{1}{2}$ " or 4"; \$0.15 each; \$1.50 dozen; \$10.00 per hundred.

*Description:* This computing device has a square cardboard base on which is mounted a circular cardboard used as a rotor. The circumference of this circle is marked according to a logarithmic scale in a manner similar to the "C" scale of an ordinary slide rule. The square has a scale matching the circle scale and is the same as the "D" scale of a slide rule. The center of the rule has a series of proportions given to show how the device can be used for multiplying, dividing, computing percentages, and solving proportions. A sheet of instructions includes directions for its use and practice problems.

*Appraisal:* This inexpensive device will probably be sufficiently durable for student use during an introductory unit on the slide rule. The simplicity of its operation using only two scales should make it suitable for junior high school pu-

pils. The scale labels every tenth subdivision between 1 and 2.5 and every half subdivision between 2.5 and 10 to aid in reading the scales. Pupils should find it very convenient to attach this square to the binders of their notebooks.

**FILMS**

*F. 102—A Day Without Numbers*

Audio-visual Materials Bureau; Wayne University; Detroit, Michigan.

B & W; 10 min.; 16 mm.; sound.

*Description:* A marionette takes a boy about and shows him what the world would be like if there were no numbers. This inspires the boy to want to learn his arithmetic.

*Appraisal:* The basic idea of this film is good, and, in general, it is well carried out. Many applications are excellent (prices, house numbers, and baseball scores), but some are doubtful in value (clock and radio dial), since they depend upon position, rather than number, and can be used without numbers at all! The grade level of the approach seems to be primary and it is suited for that age. A marionette should be interesting to that grade level, but the voice was too unusual and difficult to understand. Photographically the picture is sometimes poor: both the lighting technique and camera angles used are amateurish.

The most disturbing comments concern the intent of the film: it is confused. We are led to believe that it is supposed to motivate the learning of number combinations, and yet the only things seen are examples of the presence of numbers. This may motivate the reading, writing, or comparing of numbers, but not the addition, subtraction, or other operations. Another source of confusion, which will bother the teacher but trouble the pupil only in a vague way, is the lack of clarity on what is being magically done away with on this "day without numbers": the written number *symbol* or the *concept* of number.

*F. 103—Division of Fractions<sup>1</sup>*

Simplified Arithmetic Series, Knowledge Builders, 625 Madison Ave., New York 22, New York.

B & W; 16 mm.; sound.

*Description:* First a pie is shown and then divided into six parts. This is followed by a quick shot showing division of a whole number by a fraction, a fraction by a whole number, and a fraction by a fraction. A \$12.00 pair of skates being purchased on different partial payment plans of \$6.00 a week, or \$3.00 a week, or \$0.50 a week is used to show how the decrease in the divisor causes an increase in the quotient. The picture then shows  $12 \div \frac{1}{2}$  is  $12 \times 2/1$  and the commentator says you invert. Other illustrations used are the foot rule, part of a pie, and three quarter-pound sticks of butter. These provide the four situations with fractions. In each case the closing comment is invert and multiply.

*Appraisal:* This film seems to emphasize a process but does little or nothing to add meaning to our number system and the fundamental operations with numbers. The point most clearly presented is the inverse variation of divisor and quotient. It seems the film would tend to confuse pupils by rapid presentation of several types of exercises. The commentary and use of the pointer is not well synchronized with the frame. *Reviewed by Phillip Peak, Bloomington, Indiana.*

*F. 104—Measuring Simple Areas<sup>1</sup>*

Knowledge Builders, 625 Madison Ave., New York 22, New York.

B & W; 1 reel; 10 min.; 16 mm.; sound.

*Description:* There are some quick shots of areas in use in ancient times, both of land and buildings. The square, rectangle, parallelogram, triangle and trapezoid are shown. There is a picture of a square, one unit on a side. This unit is expanded as a

<sup>1</sup> Editorial Note: See producer's comment in "The Editor's Mail," p. 354.

unit in both directions building up rectangles of various sizes showing the area to be the product of length times width. Then follows the parallelogram with matching triangles, the area of the triangle as one-half the area of the parallelogram with the same base and altitude. The altitude and base of the triangle are held constant while the shape of the figure is changed and the area is *said* to remain constant. The area of the trapezoid is shown to be the sum of the areas of the two triangles.

*Appraisal:* This film probably does little that could not be better done with concrete materials which are easily available in most classrooms. The square of area was introduced without discussion as to its use by chance or definition. In developing the area concept, the commentator talks about unit of length while showing units of area along the sides. Although the fact that the basic area formula as  $A = bh$  was brought out, there was no mention of variability in area due to figure shape. The vocabulary was that of the tenth grade, but the film covered material which should be covered much earlier. The synchronization of sound frame and pointer was at times confusing. *Reviewed by Phillip Peak, Bloomington, Indiana.*

*F. 105—The Meaning of Plus and Minus*  
Encyclopaedia Britannica Films; 1125 Central Ave.; Wilmette, Illinois.

Color; 11 min.; 16 mm.; sound. Film guide.

*Description:* First, the teacher talks about the meaning of *three, four* and *count*. *Plus* is then introduced by means of the word *joining*; then the symbol  $+$  is shown. Many situations concerning *plus* and *minus* are discussed and contrasted using scenes in school and at a picnic. The film guide gives the story and objectives of the film; suggestions for showing, before and after; and a complete, scene by scene, description with a transcription of the sound track.

*Appraisal:* The material, speed, and vocabulary are suitable to the elementary grade levels at which the film will need to be used. The color photography and the acting are appropriate, attractive, and apt to add a great deal to motivation. The writing of numbers seen on the screen, the groups of objects, and the sound track describing the number situations are very well co-ordinated and re-inforce each other. The objects are presented in varied groups which, although motion is introduced, are clear and not confusing. One small detracting feature is the fact that the children's responses in unison are not at all easy to understand on the sound track; however, it gives a sense of reality, and the class watching the class could probably respond at the same time, correctly. This is definitely a superior film for it tries to do one, single, well-defined thing—clarify and contrast the meaning of *plus* and *minus*—and so does it well.

*F. 106—Measurement of the Speed of Light*  
McGraw-Hill Book Company, Text-film Department, 330 West 42 Street, New York 36, New York.

B & W (\$45.00); 300 feet; 8 min.; 16 mm.; sound.

*Description:* Using animated drawings we see an analysis of three methods of measuring the speed of light: Fizeau's, Foucault's, and Michelson's.

*Appraisal:* Although very useful for college physics classes it has little use for mathematics. Except for an excellent picture of four formulas, and the extremely practical geometric diagrams used throughout, there are few mathematical concepts involved.

*F. 107—Money to Loan*

Indiana University, Bloomington, Indiana, and Teaching Film Custodians, 25 West 43rd St., New York 36, New York  
Film; 16 mm, sound; Black and white; 750 feet, 20 minutes.

*Description:* In story form the film depicts the brutal methods used by unscrupulous loan sharks to collect payments

from their clients who are in arrears. There is the old man who is escorted from the factory pay-window down a lonely street by two husky toughs. They beat him into insensibility, take the amount of money due, stuff a receipt in his pocket, and leave him on the sidewalk in the dark. There is the family living in a decent residential district who find a gaudily painted panel truck equipped with bells and public address system parked in front of their home, announcing to the neighborhood that they are the type who do not pay their bills. There is the young man of conscience who informs the loan company that he will be three days late in making his next-to-last car payment. The company asks him to leave the car in the adjacent used-car lot until he gets his paycheck and returns with the payment. While he is gone, they sell his car.

A newspaper reporter sets forth to crusade against the loan sharks and rid the town of the scourge, but the loan sharks are adept at intimidating all who may speak against them. This hinders the police, who nevertheless persevere, and in a *coup* get the loan sharks and their toughs behind bars. The offended, the victims, then feel free to talk, and the loan sharks are removed from society. The town is then free of this menace.

*Evaluation:* The film is well done by Metro-Goldwyn-Mayer and utilizes recognizable name actors. The story is interesting to junior high-school students and adults as well. The film may be used in conjunction with a unit on "Borrowing Money." There is a great possibility, however, that the film does its job too well and will send the viewer away being afraid to borrow money from the many small but reputable loan agencies whose services are valuable to many individuals not able to get bank loans. *Reviewed by Robert R. Huley, Avenal, California.*

*F. 108—Uniform Circular Motion*

McGraw-Hill Book Company, Text-film Department, 330 West 42 Street, New York 36, New York.

B & W (\$45.00); 300 feet; 8 min.; 16 mm.; sound.

*Description:* This film is completely animation. It contrasts linear speed and linear velocity and derives several basic formulas from the diagram of a point moving around a circle. Vector notation and explanation is used throughout.

*Appraisal:* So many films are produced which do not require motion that it is a relief to find one where an essential part of the teaching process is the motion incorporated. This should be useful in college physics classes, but equally so in any classes which teach motion in a curve. The commentary and the quality of the animation is excellent; however, some photography of actual apparatus might help to make applications more real. There are two statements which will trouble mathematicians, although practical users in science and engineering seldom seemed troubled by them. Both are introduced by the phrase "when two points are extremely close together" and then go on to say that a line goes through the center of a circle, or that two triangles are similar. The statements are true only in a limiting position, and never for two distinct points, no matter how close together. With everything else so well done, it is too bad to find even a small slip.

#### FILMSTRIPS

*FS. 232—Learning about Using Pennies, Nickels, and Dimes*

Color (\$5.50); 40 frames; 35 mm.

*Description:* After telling the teacher what concepts are developed by this filmstrip, instructions are also given as to its proper use. Each frame pictures a series of coins with titles or questions to lead the observer to become familiar with the value of pennies, nickels, or dimes. Often the values of groups of different coins are compared. The values of the coins are also presented in terms of the objects they will buy. Problems in purchasing require simple computations with money numbers.

*Appraisal:* This filmstrip with attrac-

tive pictures of realistic purchasing situations should help build money concepts. It should not replace the use of actual coin manipulation. It would probably be most effective if actual coins were used by the pupils as they observe the strip. If it is inconvenient or impossible for the pupils to have the necessary coins, this filmstrip will be a realistic substitution.

*FS. 233—Learning to Tell Time*

Society for Visual Education, 1345 Diversey Parkway, Chicago 14, Illinois.

Color (\$5.50); 46 frames; 35 mm.

*Description:* It is the purpose of this filmstrip to teach the recognition of numbers on the face of a clock, associating number symbols and oral number words, telling time correct to the quarter-hour, and comparing the order of numbers. Each frame has a large clock face associated with an everyday situation. Each frame includes a descriptive title or question. Instructions are given to the teacher on what concepts are to be developed by the strip and how to utilize the strip for maximum effectiveness.

*Appraisal:* Each frame has a clockface that is approximately half the frame. This should furnish a clockface highly suitable for discussion and illustration. Since the classroom is usually supplied with a clock it is questionable whether it is necessary to introduce pictures of a clock for illustration.

However, the clockfaces on these frames are all associated with daily activities, most of which occur outside the classroom, and thus should build a time sense.

*FS. 234—Using and Understanding Numbers, 1-5*

*FS. 235—Using and Understanding Numbers, 5-9*

*FS. 236—Using and Understanding Numbers, 9-12*

Society for Visual Education, 1345 Diversey Parkway, Chicago 14, Illinois.

Color (\$5.50 each); 42 frames each; 35 mm.

*Description:* Each frame pictures a group of animals, people, or objects to illustrate the meaning of a given number or the groups that combine to give a certain number. Different size objects or different activities of the animals or persons are used to show the addition and subtraction combinations. Frames include the number symbol and/or a question about the number situation. The beginning of the strip gives information to the teacher of the concepts to be developed and the proper way to use the strip.

*Appraisal:* These filmstrips use attractive pictures of situations that should be of real interest to children. The purpose of the filmstrip, to have the children recognize numbers in small groups without counting, should be attainable by means of the variety of situations pictured. It is commendable that each strip confines its material to a few numbers. Even so the instruction to the teacher suggests that only part of the strip be used at a given time.

## INSTRUMENTS

### *I.42—Space Scale*

Dillon and Company, 22074 Elmwood Avenue, East Detroit, Michigan  
Scale; 5"X5"; Vinylite; \$1.50.

*Description:* The space scale is a square transparent plastic template which performs computations without the use of moving parts. It is very similar to a graph in that it has a cross-sectioned background with equal units on the vertical and horizontal reference lines. Straight and curved diagonal lines are used for reading the desired computational results. The scale can be used to multiply, divide, or extract the square root.

*Appraisal:* This scale was devised to furnish answers quickly to the engineer or builder. It can be used to obtain measurements from plans or to determine areas or volumes. According to the producer it reduces the time spent measuring areas and volumes to one-fourth the normal time. It

is a good illustration of the use of graphs to reduce repeated computations.

## MODELS

### *M. 33—Model Oil Field Kit*

Models of Industry Inc., 2804 10th Street, Berkeley 2, California.

Model Kit; 200 pieces ready to assemble; 6 drawings; booklet; teachers' manual and handbook; \$3.95.

*Description:* This kit contains the essential elements of a producing oil field, including surveying crew, aerial-mapping airplane, explosion and seismograph trucks, a drilling derrick, shothole drilling rig, gravimeter operator, replica of a pumping well and a flowing well. There are large drawings of underground strata showing what a drilling contractor must penetrate to find oil. All above surface units of the model are made to the scale of one inch to six feet and are ready to assemble. An illustrated teaching handbook gives the steps to follow in assembling the balsa wood, dowels, buttons, snaps, spools, etc., that go into the scale models. This handbook also explains each step of the exploration, drilling, production, and storage of oil. A booklet, "Story of Oil," is included and tells the story of the development of petroleum by private industry.

*Appraisal:* Although this kit was prepared for use in social studies or science there is considerable mathematics involved; for example, the plane table being used by the exploration crew, the aerial mapping by airplane, the gravimeter and seismograph, the scale representation. It will furnish appropriate material for an activity project in the upper elementary grades or junior high school where the work of several subject-matter areas is combined. Its low price is possible because of support by petroleum companies. There is no direct advertising in the printed material, but emphasis is given to the problems and operation of private enterprise in the petroleum industry.

### *M.34—Model Weather Station Kit*

Models of Industry, Inc., 2804 10th Street, Berkeley 2, California.

Model kit; pre-cut parts ready to assemble; handbook; teachers' manual; \$4.95.

*Description:* Included in this kit are most of the materials needed to construct ten weather instruments and to perform twenty-three experiments that illustrate basic weather phenomena. The kit has a manual and a handbook with complete instructions for the teacher and for the pupils, describing the materials and procedure for constructing instruments or performing experiments. This written material is prepared so that it can be taken apart easily and an individual committee given the specific section that pertains to its problem. The weather instruments that can be constructed from this kit are simple devices such as rain gauge, anemometer, air current indicator. The teachers' manual contains a bibliography of reference material and audio-visual aids.

*Appraisal:* Although this kit was prepared for elementary science classes or an aviation unit, it may be used to illustrate different types of measurement and the collection of data. It is most appropriate for an activity project in the elementary grades where the work of several subject matter areas is combined. This material was planned by a supervisor of elementary school science, Jeff B. West, so that an elementary teacher with little science background will have no difficulty teaching a unit on the weather.

### *M.35—Stadia Device*

Mr. Elwood M. Stoddard, 47A Jones Street, Hingham, Massachusetts

Kit: 4 Wood Parts; Assembly Instructions; \$0.75.

*Description:* This kit contains a 12½ inch wood bar, a slider tube, an eyepiece (large spool), a smaller spool for a handle, screws, and nails. Very simple tools and materials such as thread and adhesive

tape are needed to assemble the device. The device measures distances by means of similar triangles. The assembling of the parts and the use of the device are described simply, yet completely, in the accompanying instruction sheet.

*Appraisal:* Here is a simple, inexpensive device that can be used for inside or outside measurement projects. It can be readily assembled by the teacher or student in a matter of minutes. Although its use is limited and most measurements made are approximate, it is a clever device for illustrating the use of similar triangles.

As editors of this section since February, 1948, we wish to express our appreciation to the many persons who have contributed to its success. Many teachers have taken time to use and write evaluations; many commercial agencies have furnished materials for appraisal; and the editors of *THE MATHEMATICS TEACHER* have been very generous in furnishing space. As editors we have tried to be fair and objective in our evaluations. It is our hope that we have promoted the use of a variety of materials for more effective teaching of mathematics. It is with some regret that we terminate our monthly contribution, but at the same time we are looking forward to freedom from monthly deadlines!

In closing we would like to present the concepts of review we have tried to keep in mind.

1. Recognize that aids are only a part of the learning situation, and always subordinate to the aims and methods of the teacher.

2. Try to review both physical and mental attributes of the aid: giving suggestions for effective classroom use whenever possible.

3. Realize that every aid has some good points and some bad, but balance them to indicate general approval or disapproval.

4. Do not hesitate to review poor aids, with constructive criticisms, to warn teachers and to suggest improvement to producers.

5. Get the help of many classroom teachers so that the reviews are not purely personal opinion; place an expensive or complicated aid in classrooms and write the review after it has been used.

## • DEVICES FOR THE MATHEMATICS CLASSROOM

*Edited by Emil J. Berger, Monroe High School, St. Paul, Minnesota*

### Golden Section Compasses

*Contributed by Margaret Joseph, Milwaukee, Wisconsin*

Suppose one desires to construct a rectangle which will have pleasing proportions. What should the ratio of the sides be? The ancient Greeks' answer to this question was that the base and altitude should be in the same ratio as the two parts of a line segment divided in extreme and mean ratio. As the reader probably knows, this division is the famous *Golden Section* of a line segment, and the ratio in question is sometimes referred to as the *golden ratio*.

This brief note includes a description of a homemade device which may be used to effect the golden section of a given segment. Accordingly, whether it is appropriate or not, we have called this device a "Golden Section Compasses." As one might expect, the range of line lengths that can be divided in extreme and mean ratio with these compasses is limited by the dimensions of the device, but the principles involved are applicable for instruments of any size.

A diagram of the device is illustrated in Figure 1.  $AF$  and  $EF$  are two narrow strips of wood, each  $10\frac{1}{2}$ " long; they are joined at  $F$  with a  $\frac{1}{8}$ " machine bolt. The points  $B$  and  $D$  are so located that they divide  $AF$  and  $EF$  respectively in extreme and mean ratio—i.e.,  $AF/AB = AB/BF$ , and  $EF/FD = FD/DE$ .  $BC$  and  $CD$  are also narrow strips of wood. They are joined to each other at  $C$  and to  $AF$  and  $EF$  with  $\frac{1}{8}$ " bolts in such a way that  $BC = AB$ , and  $CD = ED$ . All measurements are from center to center.

If the device is constructed in accord-

ance with the foregoing description, it can be used to divide a line segment in extreme and mean ratio by adjusting the arms  $AF$  and  $EF$  so that  $A$  and  $E$  coincide with the extremities of the segment whose division is sought.

For definiteness, suppose that  $AE$  is a line segment which we seek to divide in extreme and mean ratio with the homemade Golden Section Compasses. If the device is adjusted and applied as suggested, then  $C$ , the intersection of  $BC$  and  $DC$ , effects the division—i.e.,  $AE/AC = AC/CE$ . That this is true is the object of the following proof:

Statements	Reasons
(1) $BC = DF = AB$ .	(1) Construction.
(2) $CD = BF = ED$ .	(2) Construction.
(3) $BCDF$ is a parallelogram.	(3) A quadrilateral whose opposite sides are equal is a parallelogram.

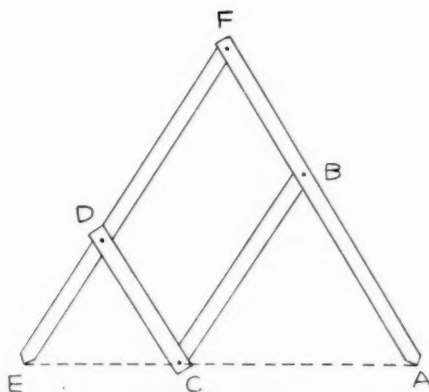


Figure 1

- |  |   |  |   |
|--|---|--|---|
| (4) $\triangle ABC \sim \triangle AFE$ . | (4) A line parallel to one side of a triangle cuts off a second triangle which is similar to the original triangle. | (7) $\frac{AB}{CD} = \frac{AC}{CE}$ .  | (7) Same reason.                        |
| (5) $\triangle ABC \sim \triangle CDE$ . | (5) If two triangles are mutually equiangular, they are similar.  | (8) $\frac{AB}{BF} = \frac{AC}{CE}$ .  | (8) Substitution (BF for CD in Step 7). |
| (6) $\frac{AE}{AC} = \frac{AF}{AB}$ .    | (6) Corresponding sides of similar triangles are proportional.  | (9) $\frac{AF}{AB} = \frac{AB}{BF}$ .  | (9) Construction.                       |
|  |   | (10) $\frac{AE}{AC} = \frac{AB}{BF}$ . | (10) Substitution (Steps 6 and 9).      |
|  |   | (11) $\frac{AE}{AC} = \frac{AC}{CE}$ . | (11) Substitution (Steps 8 and 10).     |

## A slide rule for addition and subtraction of numbers having the base 12

Contributed by Donovan A. Johnson, University of Minnesota, Minneapolis, Minnesota

Occasionally it is claimed that consideration of number systems having bases other than 10 will enhance students' understanding and appreciation of the decimal system itself. A slide rule for addition and subtraction of numbers whose base is 12 is offered here as one way of familiarizing students with the idea of numbers having a base different from the one which they normally use.

To construct such a slide rule select two cardboard strips 2"×14" and divide each with 24 equally spaced marks as shown in Figure 2. Label these division markers with the numbers of the duodecimal system from 0 to 20 and red-line the edges that are to be placed adjacent to each other when the rule is used.

Figure 2 illustrates how the rule may be used to find the sum of 7 and 8. Note that  $7+8=13$ . (base 12) In a similar way the slide can be used to find differences—i.e.,  $13-8=7$ .

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. If that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

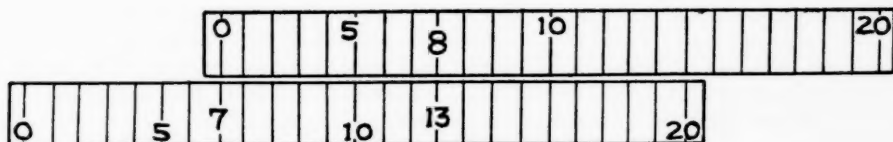


Figure 2

## • HISTORICALLY SPEAKING,—

*Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan.*

*The third and last installment on the history of complex numbers.*

*The first appeared in the February issue and the second in the April issue.*

# Complex numbers: an example of recurring themes in the development of mathematics—III

*By Phillip S. Jones*

One of the fascinating features of the study of the history of mathematics is its continual revelations of the interrelatedness of the basic elements of the subject. Some of this was displayed in our two previous notes on the history of complex numbers. The purposes of this final note are (1) to point out more of these connections, (2) to add a few comments about applications of these numbers, and (3) to do all of this with an emphasis on possible pedagogical uses of the material together with references which would aid any interested person in extending these notions much further than can or should be done here. We restrict ourselves to elementary connections with trigonometry, algebra, geometry (both synthetic and analytic), and the calculus.

In our Part II we pictured the page from Euler's *Introductio in Analysin Infinitorum* which showed exponential formulas for  $\cos \theta$  and  $\sin \theta$ . Multiplying the latter by  $i$  and adding we get the expression often referred to as Euler's formula  $e^{i\theta} = \cos \theta + i \sin \theta$ .

From this we see that we now have four related representations of complex numbers: the rectangular form,  $a+bi$ , the polar form  $r(\cos \theta + i \sin \theta)$ , the exponential form  $re^{i\theta}$ , and Wessel's graphical form named after Argand or Gauss. These relations depend on the definition that  $r = \sqrt{a^2 + b^2}$  and  $\tan \theta = b/a$ , of course. These forms serve to relate the rules for multiplying complex numbers and De-

moivre's theorem with the laws of exponents. If  $z_1 = r_1 e^{i\theta_1}$  and  $z_2 = r_2 e^{i\theta_2}$  are two complex numbers, then  $z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)} = r_1 r_2 [\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2)]$  and  $z_1^n = r_1^n e^{in\theta_1} = r_1^n (\cos n\theta_1 + i \sin n\theta_1)$ . Thus by a simple application of algebraic rules, apparently requiring no trigonometry, we have derived Demoivre's theorem and the fact that the product of two complex numbers is a complex number whose modulus is the product of the original moduli and whose amplitude is the sum of the amplitudes. Adding when multiplying is now seen to be a procedure related to complex numbers as well as to exponents and logarithms.

In this and in what follows, a rigorous treatment would require a careful sequence of definitions and theorems as well as a more detailed treatment of the limit concept and the convergence of series. This is a fact which Euler himself did not fully appreciate since modern standards of care in these matters had their beginning with Augustin Cauchy (c. 1825), in so far as any one man can be used to represent a period in the growth of ideas. Both Euler's lack of modern rigor and a usable teaching and enrichment device may be illustrated, however, by following his development a little further. Euler<sup>1</sup> set

$$(1) \quad (\cos z + i \sin z)^n = \cos nx + i \sin nx,$$

<sup>1</sup> Leonard Euler, *Introductio in Analysin Infinitorum* (Lausanne, 1748) p. 97ff. Also to be found in *Leonhardi Euleri Opera Omnia* (Lipsiae et Berolini, 1922), Series Prima, VIII, 140 ff.

expanded the left member by the binomial theorem, collected the real and imaginary parts and equated them to  $\cos nz$  and  $\sin nz$  respectively. Today this is a convenient way to derive or remember the multiple angle formulas. For example, taking  $n=3$  we have

$$\begin{aligned} & (\cos^3 z - 3 \cos z \sin^2 z) \\ (2) \quad & + i(3 \cos^2 z \sin z - \sin^3 z) \\ & = \cos 3z + i \sin 3z. \end{aligned}$$

Substituting for  $\sin^2 z$  we readily derive  $\cos 3z = 4 \cos^3 z - 3 \cos z$ , a well-known identity. Incidentally, this, by letting  $3z = 60^\circ$ , leads to

$$(3) \quad 8x^3 - 6x - 1 = 0,$$

a cubic whose root  $x = \cos 20^\circ$  is not "constructable" and thus furnishes one proof of the impossibility of trisection under Plato's restrictions. An unlimited number of trigonometric formulas may be "derived" this way. For example, if

$$z_1 = 1(\cos \alpha + i \sin \alpha)$$

and

$$z_2 = 1(\cos \beta + i \sin \beta),$$

then

$$\begin{aligned} z_1 \cdot z_2 &= \cos(\alpha + \beta) + i \sin(\alpha + \beta) \\ (4) \quad &= (\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\ &+ i(\sin \alpha \cos \beta + \cos \alpha \sin \beta) \end{aligned}$$

from which by equating real and imaginary parts we derive formulas for  $\cos(\alpha + \beta)$  and  $\sin(\alpha + \beta)$ . (Actually, of course, most modern developments, as well as Euler's, use these identities to derive the multiplication rule for complex numbers, and then obtain De Moivre's theorem from it by induction. However, once this groundwork is laid, these complex number techniques are then a valid source of further trigonometric identities as well as a mnemonic device for over-learning, or recalling the basic formulas.)

Euler derived from (1)

$$\begin{aligned} \sin nz &= \frac{n}{1} (\cos z)^{n-1} \sin z \\ &+ \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} (\cos z)^{n-3} (\sin z)^3 \\ &+ \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} (\cos z)^{n-5} (\sin z)^5, \text{ etc.} \end{aligned} \quad (5)$$

He then stated: "If the arc  $z$  is infinitely small,  $\sin z = z$  and  $\cos z = 1$ ; moreover, if  $n$  is a number infinitely large, so that the arc  $nz$  is of finite magnitude, think  $nz = v$  or  $\sin z = z = v/n$  and then

$$(6) \quad \sin v = v - \frac{v^3}{1 \cdot 2 \cdot 3} + \frac{v^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}, \text{ etc.}"$$

Euler's procedure is clear and usable with young students and for club programs and projects if one modernizes his statement to read

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1, \quad \lim_{\theta \rightarrow 0} \cos \theta = 1$$

and then in place of (1) writes

$$(7) \quad \left( \cos \frac{x}{n} + i \sin \frac{x}{n} \right)^n = \cos x + i \sin x.$$

This would lead to

$$\begin{aligned} \sin x &= n \left( \cos \frac{x}{n} \right)^{n-1} \left( \sin \frac{x}{n} \right) \\ &+ \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \left( \cos \frac{x}{n} \right)^{n-3} \left( \sin \frac{x}{n} \right)^3 \\ &+ \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \left( \cos \frac{x}{n} \right)^{n-5} \left( \sin \frac{x}{n} \right)^5 \\ &\text{etc.} \end{aligned} \quad (8)$$

Multiplying the numerator and denominator of the first term on the right by  $x/n$ , of the second by  $(x/n)^3$ , the third by  $(x/n)^5$ , etc., we have, after distributing the factors properly,

$$\begin{aligned} \sin x = & x \left( \cos \frac{x}{n} \right)^{n-1} \left\{ \frac{\sin \frac{x}{n}}{\frac{x}{n}} \right\} - \frac{1 \left( 1 - \frac{1}{n} \right) \left( 1 - \frac{2}{n} \right) x^3}{1 \cdot 2 \cdot 3} \left( \cos \frac{x}{n} \right)^{n-3} \left\{ \frac{\sin \frac{x}{n}}{\frac{x}{n}} \right\}^3 \\ & + \frac{1 \left( 1 - \frac{1}{n} \right) \left( 1 - \frac{2}{n} \right) \left( 1 - \frac{3}{n} \right) \left( 1 - \frac{4}{n} \right) x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \left( \cos \frac{x}{n} \right)^{n-5} \left\{ \frac{\sin \frac{x}{n}}{\frac{x}{n}} \right\}^5, \text{ etc.} \end{aligned} \quad (9)$$

Now taking the limit as  $n \rightarrow \infty$  and  $x$  remains fixed we have Euler's result, the standard series expansion for  $\sin x$ . Although lacking in rigor (for example, it assumes the series converges, converges to  $\sin x$ , and that the limit of a sum is the sum of the limits), this procedure arrives at the sine series without, in a sense, the use of calculus. It introduces the limit idea easily, it associates trigonometry, De Moivre's theorem, and the binomial expansion, and, historically, it illustrates the ingenuity of Euler and an interesting stage in the development of both trigonometry and calculus.

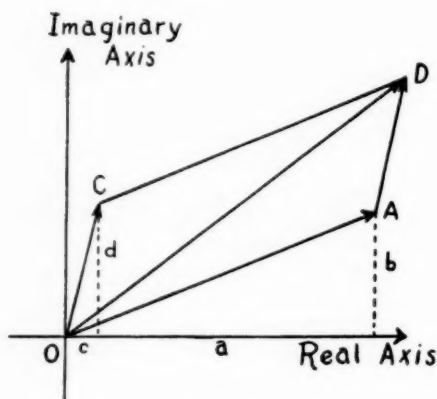


Figure 1

Many of the applications of complex numbers, both mathematical and physical, stem from the fact that they can be interpreted as an algebraic counterpart of vectors. Thus the complex number

$z_1 = a + bi = r(\cos \theta + i \sin \theta)$  can be associated with the point  $(a, b)$  or the family of vectors parallel and equal in length to the line from  $O$  to  $A \equiv (a, b)$  as in Figure 1.

The sum of  $z_1 = a + bi$  and  $z_2 = c + di$  corresponds to the vector  $\vec{OD}$  which is the diagonal of the parallelogram determined by  $\vec{OA}$  and  $\vec{OC}$ . This can also be regarded as the third side,  $OD$ , of a triangle  $OAD$  where the sum vector of  $z_1 + z_2$  is that vector which has its "tail" at the tail,  $O$ , of  $z_1$  and its "head" at the head,  $D$ , of  $z_2 = \vec{AD}$  where  $\vec{AD}$  is parallel and equal to  $z_2 = \vec{OC}$ . The vector equal and parallel to vector  $OD$  but oppositely directed (Figure 2) is denoted by  $\vec{DO}$  and is defined as the

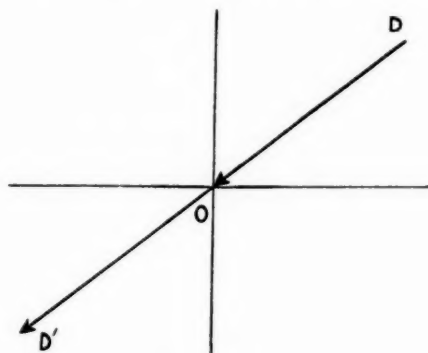


Figure 2

negative of  $\vec{OD}$ , i.e.  $\vec{DO} = -\vec{OD} = \vec{OD}'$  where  $\vec{OD}'$  has the same length and direction as  $\vec{DO}$ .

This is equivalent to stating that if the vector representations of three complex numbers can, without changing their lengths or directions, be slid ("translated") in such a way as to form a triangle when joined "tail to head," then the sum of the numbers (vectors) is zero. Many facts in plane geometry, mechanics, and trigonometry can be derived from this. For example, to derive the *sine law* consider the general triangle  $ABC$  (Figure 3) with its side  $AB$  located along the real axis. If its sides are thought of as vectors to be represented as complex numbers we then have

$$\vec{AB} = c(\cos 0^\circ + i \sin 0^\circ),$$

$$\vec{BC} = a[\cos(180^\circ - B) + i \sin(180^\circ - B)],$$

$$\vec{CA} = b[\cos(180^\circ + A) + i \sin(180^\circ + A)].$$

Since this is a closed triangle

$$\begin{aligned} \vec{AB} + \vec{BC} + \vec{CA} &= 0 \\ (10) \quad &= (c - a \cos B - b \cos A) \\ &\quad + i(a \sin B - b \sin A). \end{aligned}$$

Equating the imaginary part to zero we have

$$\frac{\sin A}{a} = \frac{\sin B}{b}.$$

Using this result and that obtained by setting the real part of (10) equal to zero, one can next derive the Law of Cosines.<sup>2</sup>

Turning from trigonometry to geometry we would like to indicate that all of synthetic and analytic plane geometry can be neatly done using complex number-vector

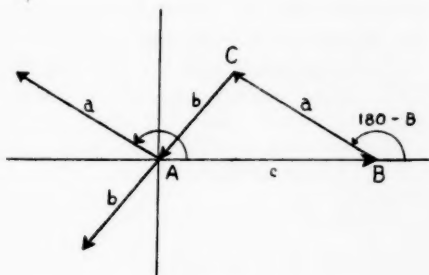


Figure 3

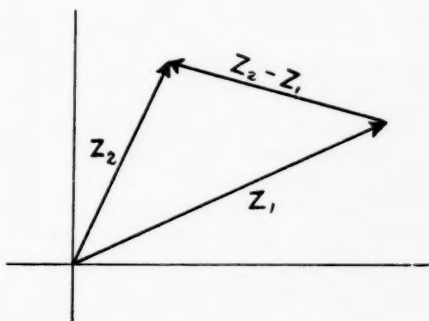


Figure 4

ideas. Our contention here, however, is not that these subjects should be taught in this manner, but that turning to these ideas occasionally in class or in a club program will make complex numbers both more real and more fascinating as well as clarifying and broadening the students' views of geometry.

To do this let's add these further notions.

- (11)  $z_2 - z_1$  is the vector from the head of  $z_1 = x_1 + iy_1$  to the head of  $z_2 = x_2 + iy_2$  as shown in Figure 4.
- (12)  $k \cdot z_1$ , where  $k$  is a real number and  $z_1$  is complex, represents a vector parallel to  $z_1$  and  $k$  times as long.

With these tools we will prove one familiar theorem as an example of what can be done.

**THEOREM:** The line joining the midpoints of two sides of a triangle is parallel to and equal to one half of the third side.

For convenience choose the axes such that vertex  $A$  is at the origin and side  $AC$

<sup>2</sup> Edward M. J. Pease and George Wadsworth, *Engineering Trigonometry* (Scranton, Pa.: International Textbook Co., 1946) contains this and similar derivations together with good discussions of the use of complex numbers in electrical engineering and in geometric inversion which in turn is applied to electrical engineering.

along the real axis (Figure 5). Then if  $z_1 = \overrightarrow{AC}$  and  $z_2 = \overrightarrow{AB}$ , then  $z_2 - z_1 = \overrightarrow{CB}$ .

The midpoints  $M$  and  $N$  of  $AB$  and  $CB$  are then the ends of vectors

$$\overrightarrow{AM} = \frac{1}{2}z_2 \quad \text{and}$$

$$\overrightarrow{AN} = z_1 + \overrightarrow{CN} = z_1 + \frac{1}{2}(z_2 - z_1) = \frac{1}{2}(z_2 + z_1).$$

The line  $MN$  then corresponds to the vector

$$\overrightarrow{AN} - \overrightarrow{AM} = (\frac{1}{2}(z_2 + z_1) - \frac{1}{2}z_2).$$

Q.E.D.

It would be quite simple to add concise complex number (vector) formulas for the distance between two points, the angle between two lines, parallelism and perpendicularity and thence to go on to all Euclidean geometry. The references given will help you to do so.<sup>3</sup> We leave the field of geometry by adding that in addition to this interesting but quite unnecessary approach to Euclidean geometry there is an extensive geometry of the complex plane as well as important uses of complex numbers in projective and inversive geom-

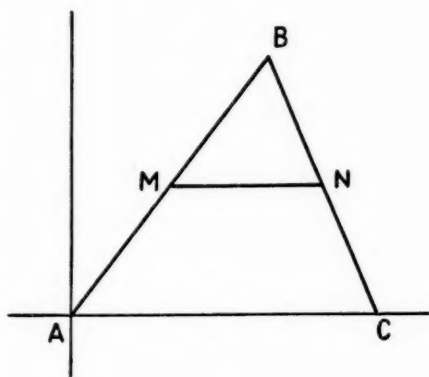


Figure 5

etry.<sup>4</sup> However, before leaving geometry, we would like to note that the problem of what regular polygons are constructible, which had its origins in Greek geometry as far back as the mystic pentagram of the Pythagoreans, had its final solution in algebraic terms at the hands of C. F. Gauss in 1799. One interesting and elementary approach to this problem is via the  $n$  roots of unity and Demoivre's theorem, another mathematical application of complex numbers.<sup>5</sup>

Turning now to the theory of equations, where complex numbers had their birth, there are interesting elementary schemes both for representing complex roots and

<sup>3</sup> Howard Fehr, *Secondary Mathematics* (Boston: D. C. Heath & Co., 1951) contains an excellent but slightly different presentation pp. 255-58, 292 ff. This book also contains a discussion of the graphical interpretation of the multiplication and division of complex numbers, and series developments such as we outlined. Here, too, it is made clear that complex numbers make it possible to have logarithms of negative numbers, but, unfortunately, the author does not point out that we may also have (complex) "angles" whose sines are greater than 1.

Allen A. Shaw, "Geometric Applications of Complex Numbers," *School Science and Mathematics*, Vol. 31 (1931), pp. 754-61.

Allen A. Shaw, "Applications of Complex Numbers to Geometry," *THE MATHEMATICS TEACHER*, XXV (1932), 215-26.

Allen A. Shaw, "Applications of Complex Numbers to the Geometry of Circles," *National Mathematics Magazine*, Vol. 14 (1939), pp. 26-35.

Our approach is modeled after Shaw's who in turn followed Smail and Schelkunoff.

Lloyd L. Smail, "Some Geometric Applications of Complex Numbers," *American Mathematical Monthly*, Vol. 36 (1929), pp. 504-11. Professor Smail's *College Algebra* also contains some of this material.

G. A. Schelkunoff, "A Note on Geometrical Applications of Complex Numbers," *American Mathematical Monthly*, Vol. 37 (1930), pp. 301-3.

<sup>4</sup> J. L. Coolidge, *Geometry of the Complex Domain* (Oxford: 1924).

C. Zwikker, *Advanced Plane Geometry* (Amsterdam, 1950). Available through Interscience Publishers Inc., New York, this book, written by an electrical engineer, uses  $j$  for  $i$  and includes many applications in electrical theory but also treats "modern" geometry, conics, spirals, and gear-wheel profiles, as a few of many topics. It is not really an elementary text, however.

Emile Borel and Robert Deltheil, *La Géométrie et les Imaginaires* (Paris, 1931).

William C. Graustein, *Introduction to Higher Geometry* (New York: The Macmillan Co., 1935), chap. viii, and also pp. 145 ff., 396-400.

<sup>5</sup> F. Klein (edited by R. C. Archibald), *Famous Problems of Elementary Geometry* (New York: Hafner Publishing Co., 1950), chap. iii.

R. C. Yates, *Geometrical Tools* (St. Louis: Educational Publishers).

Felix Klein, *Elementary Mathematics from an Advanced Standpoint*, Vol. I (New York: Dover Publications, 1945), p. 45 ff. First Part, chap. iv.

for solving for them.<sup>6</sup> We will add here only a note on a well-known fact not covered by our references; namely, since the vertex of the parabola  $y = ax^2 + bx + c$  is at  $[-(b/2a), -(b^2 - 4ac)/4a]$ , the complex roots of a quadratic may be read from a graph as  $A \pm Bi$  where  $A$  is the abscissa of the vertex of the parabola and  $B$  is the square root of the quotient of its ordinate divided by  $a$ . If the leading coefficient is taken to be 1, the division is unnecessary, and in either case  $B$  could be constructed with ruler and compasses if one wished to be still more geometric. Finally, don't fail to point out to students that just as complex numbers made possible the theorem that the  $N$ th degree equation has  $N$  roots, so they also made the factoring of these polynomials possible; e.g.  $x^2 + a^2 = (x + ai)(x - ai)$ .

Let us now turn to the physical applications of complex numbers as distinct from applications in developing further mathematics (which latter, however, may be important for physical applications as well as intrinsically). We noted in our earlier chapter their use in calculating capac-

itances and resistances. An unusually interesting article on this subject by Edward M. Noll appeared under the title "Meet Mr. j!" in *Q.S.T.*, the journal of the Amateur Radio Relay League, Volume XXVII (October 1943).<sup>7</sup> This re-emphasizes the facts that mathematics may be important even in leisure-time activities and to nontechnically trained persons, and that in electrical work  $i = \sqrt{-1}$  is replaced by  $j$  since  $i$  has another meaning there. Other notations for complex numbers in common use include  $r \text{ cis } \theta$  and  $r/\theta$  for  $r(\cos \theta + i \sin \theta)$ .

Our last set of references<sup>8</sup> includes introductory discussions of the theory of functions of a complex variable which in turn include illustrations drawn from many fields in which this theory plays an important role. Some of these are: aerodynamics, hydrodynamics, mapmaking, elasticity, theory of potential, heat flow, electrostatic flux, the study of periodic phenomena.

<sup>6</sup> Interesting representations of complex roots are discussed in Fehr, *op. cit.*, pp. 285-292, and more technically in Luise Lange, "On a Three Dimensional Presentation of Functions of a Complex Variable," *American Mathematical Monthly*, Vol. 46 (1939), pp. 190-98.

A further discussion of this type with models and a method for constructing the roots of a quadratic is to be found in Howard F. Fehr, "Graphical Representation of Complex Roots," in *Multi-Sensory Aids in the Teaching of Mathematics*. Eighteenth Yearbook (The National Council of Teachers of Mathematics, New York: 1945), pp. 130-38.

Elementary and geometric schemes for solving for the complex roots of equations may be found in: Fehr, *ibid.*

H. M. Gehman, "Complex Roots of a Polynomial Equation," *American Mathematical Monthly*, Vol. 48 (1941), p. 237 ff.

George A. Yanosik, "A Graphical Solution for the Complex Roots of a Cubic," *National Mathematics Magazine*, Vol. 10 (1936), p. 139 ff.

H. T. R. Aude, "A Note to the Theory of Equations," *National Mathematics Magazine*, Vol. 14 (1940), p. 308 ff.

T. R. Running, "Graphical Solutions of Cubic Quartic and Quintic," *American Mathematical Monthly*, Vol. 50 (1943), pp. 170 ff.

T. R. Running, *Graphical Mathematics* (New York: John Wiley & Sons, Inc., 1927), p. 34 ff.

John J. Corliss, "The Solution of Quadratic Equations by Means of Complex Numbers," *School Science and Mathematics*, Vol. 38 (1938), pp. 256-58.

<sup>7</sup> Further discussions of and approaches to these applications may be found in: Fred Gruenberger, "Imaginarities," *THE MATHEMATICS TEACHER*, XLVII (1954), 11 ff.

"Nomographic Evaluation of Complex Numbers," *Radio News*, Vol. 31 (March 1944), p. 29.

*Q.S.T.*, Vol. 27 (October 1943), p. 21 gives the familiar graphical device for solving  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ .

This may also be found in "A Graphical Solution of Some Common Problems," by John Colliton in *THE MATHEMATICS TEACHER*, XXXIX (1946), 391.

<sup>8</sup> Ruel V. Churchill, *Introduction to Complex Variables and Applications* (New York: McGraw-Hill Book Co., Inc., 1948). This book has a good short bibliography of works on function theory and its applications.

R. W. Reddick and F. H. Miller, *Advanced Mathematics for Engineers* (New York: John Wiley & Sons, Inc., 1938), chap. x.

Theodore V. Karman and Maurice A. Biot, *Mathematical Methods in Engineering* (New York: McGraw-Hill Book Co., Inc., 1940), chap. ix, "Complex Representation of Periodic Phenomena."

#### CORRECTION

In the March, 1954, issue of *THE MATHEMATICS TEACHER*, page 195, the number 4473 was incorrectly represented. The correct representation is MC LXXIII.

□ □ □

## • MATHEMATICAL MISCELLANEA

*Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and  
Adrian Struyk, Clifton High School, Clifton, New Jersey.*

*Squares—geometric and arithmetic.*

### *The Pythagorean theorem— proof number one thousand*

*Contributed by J. C. Eaves, Alabama Polytechnic Institute, Auburn, Alabama*

#### 1. INTRODUCTION

The writer has found students at various levels who had no clear understanding of the meaning of the Pythagorean theorem. This was brought out in a course, "The Fundamentals of Algebra," given for in-service teachers, and the professor was immediately confronted with the task of presenting or submitting to these teachers some method of demonstration which would prove more effective than the ordinary approach. The following is one of the several proofs and demonstrations which resulted from this discussion.

One advantage in this approach seems to be that a clear demonstration of the theorem can be made and then, by using geometric constructions, a formal proof can be given, based upon the same general idea of the demonstration. We may also note that the proof is completely general and may be made either algebraic or geometric. The construction is the type which Loomis<sup>1</sup> refers to as "The h(ypotenuse)—square constructed interior."

#### 2. DEMONSTRATION GADGET

Cut from cardboard or light plastic sheets, the necessary triangles, squares, etc., as shown in Figure 1 and pivot vertices so that gadget can be unrolled as in Figure 2 and continue until Figure 3 is ob-

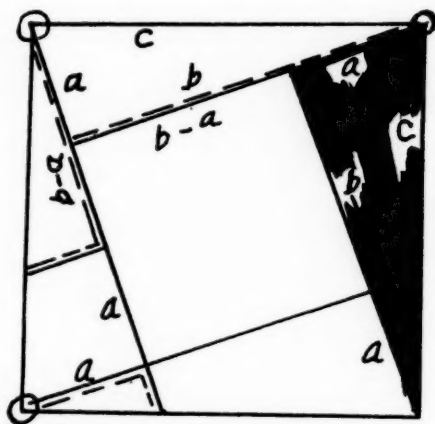


Figure 1

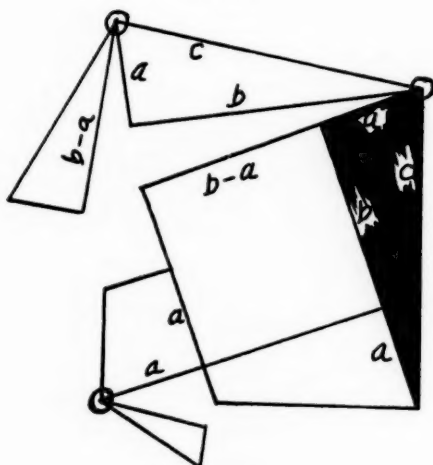


Figure 2

<sup>1</sup> Elisha S. Loomis, *The Pythagorean Proposition* (Edwards Bros., 1940), p. 144. This book contains 370 demonstrations of this theorem.

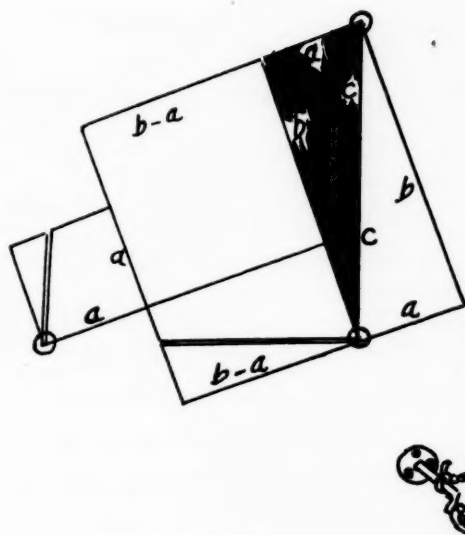


Figure 3

tained. In Figure 1 the large square, whole model, is the square which is constructed on the hypotenuse of the basic triangle (in black). In Figure 3 the model has been pivoted into two squares, one of the size which could be constructed on the long leg of the basic triangle, the second on the short leg of the basic triangle. Thus the demonstration is completed. (The special case where the basic triangle is isosceles-right gives a surprising gadget.)

### 3. ALGEBRAIC PROOF

Variations of the following algebraic proof have been attributed to many people. Using Figure 1, the hypotenuse square is equal to the sum of the four (congruent) triangles and the inner square. From this, it follows that  $c^2 = 4(ab/2) + (b-a)^2$ , which reduces to  $c^2 = a^2 + b^2$ .

### 4. GEOMETRIC PROOF

Beginning with Figure 1, as basic, make the constructions as indicated by dotted lines in Figure 5. It is easy to prove that  $\Delta 1 = \Delta 1'$ ,  $\Delta 2 = \Delta 2'$ ,  $\Delta 3 = \Delta 3'$ ,  $KL$  and  $MN$  are squares, and therefore  $\square PQ = \square PQ + \Delta 1 - \Delta 1' + \Delta 2 - \Delta 2'$

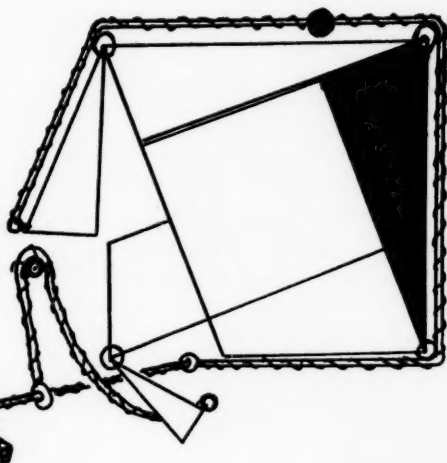


Figure 4

$+ \Delta 3 - \Delta 3' = \square KL + \square MN$  and the theorem is established.

### 5. TURN-THE-CRANK DEMONSTRATION

This model has been built of large size for classroom use and of rather heavy material ( $\frac{1}{4}$ " plywood or wallboard) and mounted on a 3' by 3' section of the same material. A very novel effect is achieved by attaching strings and screw eyes as indicated in Figure 4 and the theorem is "proved" in a manner sought by many—just turn the crank.

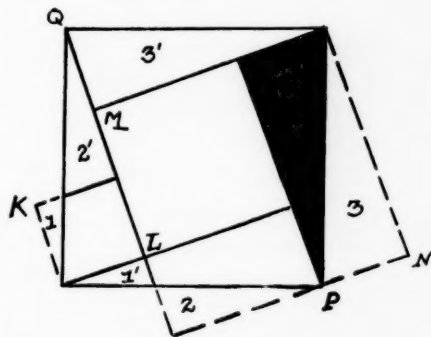


Figure 5

# On numbers which terminate perfect squares<sup>1</sup>

Contributed by Victor Thébault, Tennie, Sarthe, France

I. Since the product  $a(a+1)$  is even the square

$$(1) \quad (2a+1)^2 = 4a^2 + 4a + 1 = 4a(a+1) + 1$$

of an odd integer is simultaneously a multiple of 4 and of 8 increased by unity. Hence, with zero as a permissible value of  $d$ , an odd perfect square necessarily ends in a two-digit number  $d1$ ,  $d5$ ,  $d9$  such that

$$10d = 4p, \quad 10d + 4 = 4p, \quad 10d + 8 = 4p.$$

Then the integers  $5d$ ,  $5d+2$ ,  $5d+4$  are even, and therefore too the tens digit  $d = 2d'$  of every odd perfect square.

COROLLARIES. 1. If the units digit of a square is 1 or 9, the hundreds digit,  $c$ , and half of the tens digit,  $d' = \frac{1}{2}d$ , are of like parity.<sup>2</sup>

PROOF: Since any multiple of 1000 is divisible by 8 the three-digit number which terminates the square is

$$\begin{aligned} cd1 &= 100c + 10d + 1 \\ &= 100c + 20d' + 1 = 8p + 1. \end{aligned}$$

Then we may write in succession

$$\begin{aligned} 100c + 20d' &= 8p, \\ 25c + 5d' &= 2p, \\ 2(12c + 2d') + c + d' &= 2p. \end{aligned}$$

Therefore  $c+d'$  is necessarily even, and  $c$ ,  $d'$  are of like parity. Similarly, if the three-digit number  $cd9$  terminates a square we may write

$$\begin{aligned} 100c + 20d' + 9 &= 8p + 1, \\ 25c + 5d' + 2 &= 2p, \\ 2(12c + 2d' + 1) + c + d' &= 2p, \end{aligned}$$

<sup>1</sup> Translated from the French by Adrian Struyk.

<sup>2</sup> This proposition is demonstrated less simply in Fitz-Patrick: *Exercices d'Arithmétique*, p. 488, question 635.

again making  $c$  and  $d'$  alike in parity.

2. If a perfect square of the form  $16p+1$  ends with a four-digit number  $mcd1$ , then the thousands digit  $m$  and the digit representing  $\frac{1}{2}(c+d')$  are of like parity; if it ends with  $mcd9$  then  $m$  and  $\frac{1}{2}(c+d')$  are of opposite parity.

PROOF: Since any multiple of 10,000 is divisible by 16 we may write for  $mcd1$

$$\begin{aligned} 1000m + 100c + 20d' &= 16p, \\ 1000m + 96c + 16d' + 4c + 4d' &= 16p, \\ 125m + 12c + 2d' + \frac{1}{2}(c+d') &= 2p, \\ 2(62m + 6c + d') + m + \frac{1}{2}(c+d') &= 2p. \end{aligned}$$

Hence  $m$  and  $\frac{1}{2}(c+d')$  agree in parity. Similarly, for  $mcd9$  we may write

$$1000m + 100c + 20d' + 8 = 16p,$$

which reduces to

$$2(62m + 6c + d') + m + \frac{1}{2}(c+d') + 1 = 2p.$$

Therefore  $m + \frac{1}{2}(c+d')$  is necessarily odd, so that  $m$  and  $\frac{1}{2}(c+d')$  are of opposite parity.

3. If a perfect square of the form  $32p+1$  ends in a five-digit number  $tmed1$ , then the ten-thousands digit  $t$  and half of the tens digit,  $d'$ , are of like or opposite parity according as the number  $\frac{1}{2}[m + \frac{1}{2}(c+d')]$  is even or odd, if it ends in  $tmed9$  then  $t$  and  $d'$  agree in parity or not according as  $\frac{1}{2}[m + \frac{1}{2}(c+d') + 1]$  is even or odd.

PROOF: The first part results from the equation

$$10000t + 1000m + 100c + 20d' = 32p.$$

This may be written as

$$\begin{aligned} 10000t + 992m + 96c + 16d' \\ + 8m + 4c + 4d' = 32p. \end{aligned}$$

Dividing both members by 16,

$$625t + 62m + 6c + d'$$

$$+ \frac{1}{2} [m + \frac{1}{2}(c + d')] = 2p.$$

Finally, letting  $K = 312t + 31m + 3c$ ,

$$2K + t + d' + \frac{1}{2} [m + \frac{1}{2}(c + d')] = 2p,$$

and the conclusion follows.

In the second case, beginning with

$$10000t + 1000m + 100c + 20d' + 8 = 32p,$$

the final result becomes

$$2K + t + d' + \frac{1}{2} [m + \frac{1}{2}(c + d') + 1] = 2p.$$

*Examples.* Cor. 1.  $41^2 = 1681$ ;  $c = 6$ ,  $d' = 4$ .

$$89^2 = 7921$$
;  $c = 9$ ,  $d' = 1$ .

Cor. 2.  $79^2 = 6241$ ;  $m = 6$ ,  $\frac{1}{2}(c + d') = 2$ .

$$127^2 = 16129$$
;  $m = 6$ ,  $\frac{1}{2}(c + d') = 1$ .

Cor. 3. Let  $S$  denote  $m + \frac{1}{2}(c + d')$ .

$$159^2 = 25281$$
;  $t = 2$ ,  $d' = 4$ ,  $\frac{1}{2}S = 4$ .

$$191^2 = 36481$$
;  $t = 3$ ,  $d' = 4$ ,  $\frac{1}{2}S = 5$ .

$$257^2 = 66049$$
;  $t = 6$ ,  $d' = 2$ ,  $\frac{1}{2}(S + 1) = 4$ .

$$513^2 = 263169$$
;  $t = 6$ ,  $d' = 3$ ,  $\frac{1}{2}(S + 1) = 3$ .

*Note.* Knowledge of the above relationships is useful in determining, *a priori*, whether or not a number ending in 1 or 9 is a perfect square.

II. Equalities such as (1) occur in any system of numeration of base  $B$  consistent with the following remarks.

(a) Suppose that the square of an odd number, written in the base  $B$ , ends in a two-digit number of the form

$$(2) \quad B \cdot d + (2u + 1) = 4p + 1.$$

If  $B$  is odd then, by (2), the penultimate digit  $d$  represents an even number. When the two-digit terminus of the square is even then

$$B \cdot d + (2u + 1) = 2p',$$

and  $d$  is of like parity with  $B$ , that is to say odd.

*Examples.* Take  $B = 11$ , and let  $T = B - 1$ .

$$16^2 = 243, \quad 67^2 = 4005;$$

$$34^2 = 1035, \quad 77^2 = 7315.$$

(b) Suppose that an odd square written in base  $B = 2(2n + 1)$  ends in a two-digit number of the form

$$B \cdot d + (4u + 1),$$

that is,

$$(2n + 1)d + 2u = 2p.$$

Then  $d$  is even for all odd squares of this form in these bases. This is the case where  $n = 2$  and  $B = 10$  considered at the beginning.

(c) When an odd square written in base  $B = 4(2n + 1)$  ends in a two-digit number  $B \cdot d + (8u + 1)$ , we get

$$(2n + 1)d + 2u = 2p.$$

Hence  $d$  is even for all odd squares of this form in this base.

*Examples.* Take  $n = 1$  and  $B = 12$ .

$$3^2 = 09, \quad 5^2 = 21, \quad 7^2 = 41,$$

$$9^2 = 69, \text{ and } E^2 = T1, \text{ where}$$

$$E = B - 1 \text{ and } T = B - 2.$$

*N.B.* The only bases of types (b) and (c) which possess the property that  $d$  is always even for all odd squares written in these bases are

$$(b) \quad B = 2, 10; \quad (c) \quad B = 4, 12.$$

For there are no others in which we do not meet with perfect squares formed exclusively by odd digits.<sup>3</sup>

Part I of this note appeared also in *Mathesis* (1953).

<sup>3</sup> Thébault, V. *Les Récréations Mathématiques (Parmi les nombres curieux)* p. 137. (Gauthier-Villars, Paris, 1952).

## ● REFERENCES FOR MATHEMATICS TEACHERS

Edited by William L. Schaaf, Department of Education,  
Brooklyn College, New York

### Mathematics and people

Under this somewhat unusual title I have deliberately brought together material on several distinct aspects of the relationship of people, in general, to mathematics. It is fairly common to regard mathematics as devoid of social interest, as something impersonal; indeed, almost unhuman in its aloofness. Yet the connections between mathematics and society, not always obvious, are none the less real and significant.

#### 1. THE SOCIOLOGY OF MATHEMATICS

D. J. Struik has suggested that the sociology of mathematics concerns itself with the influence of forms of social organization on the origin and growth of mathematical conceptions and methods, and the role of mathematics as part of the social and economic structure of a period. He goes on to say that "the primitive forms of society, the oriental, the Greco-Roman, the medieval feudal, the early capitalistic, the modern capitalistic and the socialist forms of society have all influenced in their various ways the acquisition of mathematical knowledge and have been in their turn subjected to its influence." While it is undoubtedly true that sociological considerations have influenced the development of both science and mathematics, the actual nature of this influence is not yet clearly understood. Comparatively little research has been undertaken along these lines, and there would seem to be considerable room for a variety of interpretation. According to Oswald Spengler, for example, the soul

of a culture is reflected in the kind of mathematics it produces. Professor Struik, while conceding that mathematical activity cannot be attributed to any single cause, is disposed, possibly, to overemphasize the impact of economic forces in shaping the nature and direction of mathematical development. There are those who believe in the validity of applying the method of dialectical materialism to all scientific disciplines—including mathematics. On the other hand, George Sarton, admitting that mathematical creations and discoveries are shaped to some extent by social forces and external circumstances, insists that the main source of mathematical activity lies within individual men themselves—their creative genius, their insatiable curiosity, indeed, even their poetic caprice.

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- Human relations and social concepts, extremely complex at best, are at once both qualitative and quantitative in nature. One might understandingly be tempted to object that economics, political science, history, and demography, bristling with motives, values, opinions, judgments, emotions and personalities, hardly admit of sober mathematical analysis. On the contrary. Social phenomena can be studied in two ways: from the descriptive, empirical standpoint—the methods of statistics and probability—or as the systematic development of a general mathematical theory from which quantitative relations may be derived. The former approach is, of course, an old story. The latter method has long been used in the development of economic theory, and in recent years has been successfully applied in the fields of sociology and human biology, following the brilliant pioneer work of Nicolas Rashevsky.
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### 3. MATHEMATICS AND THE SOCIAL SCIENCES

The applications of mathematical methods, both empirical as well as analytic, to the social sciences generally are suggested in Sections 3 and 4. They include a variety of problems which touch upon the daily lives of all people—population trends, taxes, incomes, civic matters, elections, voting, opinion polls, apportionment, trade unions, cost of living, social security, employment, commerce, manufacturing, investment and finance, public services and utilities, international trade—endless ramifications of the relations between people and their activities. The references below are merely suggestive; the literature is enormous.

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#### 4. POLITISCHE ARITHMETIK; MATHEMATISCHE VOLKSWIRTSCHAFTS-LEHRE UND STAATSBÜRGERKUNDE

These few references, albeit out of date and largely inaccessible to American readers, have nevertheless been included to suggest a type of material which could properly and profitably be made available to our young people. Material on the mathematics of citizenship, of the community, of everyday business, of household and personal finance, of the job, and of the market place should be far more widely distributed and taught than at present.

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## The editor's mail

It may appear a bit unusual for the producers of the two films, "Measuring Simple Areas" and "Division of Fractions," to ask each and every reader of the appraisal to send us their name and address with a request to preview the above films under classroom conditions and then to poll their class for favorable or unfavorable comments with regard to the teaching content.

All of teaching procedure of the films in the Plane Geometry Series and in the Arithmetic Series was prepared by Ray C. Jurgensen of the mathematics staff of Culver Military Academy. Mathematics and particularly geometry is given first importance in military schools. All of these films have been selected to be translated into French and Spanish by two foreign governments.

There has long been a crying need for good sound films to help visualize the concepts of mathematics. During the past nine years we have spent approximately \$100,000 to bring useful mathematics films to the classrooms of America. If we have in any measure failed in this effort then there is certainly a golden opportunity awaiting the person who can produce the ideal films.

(signed) KNOWLEDGE BUILDERS  
JOHN R. McCORRY  
John R. McCorry, Director

It is high time that I express my appreciation to the editorial staff of our fine national magazine for the three years of enjoyable reading I have received, and to compliment you on the "new look" given to the magazine, and to say that I will continue to look forward eagerly to each issue.

Recently, a third-semester algebra student of mine "discovered" for himself a formula that can be used to facilitate work with certain radical expressions. We had introduced fractional exponents and, among other problems, I had assigned the problem  $\sqrt{2} \cdot \sqrt[3]{3}$ . They had mastered—I use the word loosely—the cases of like bases and like indices, but this case was new to them. Since it caused difficulty, I demonstrated the technique of changing to fractional exponents with a least common denominator,

etc. As I was concluding, the student, Jim Hudson by name, asked if we could not obtain the answer more simply. He had thought of a rule, which he explained, we converted his notion into symbolic form, proved him to be correct, and immediately added it to a list of "laws of exponents" which we had been compiling. It was given the name, "Hudson's Rule," and this is it.

$$\sqrt[n]{x^a} \cdot \sqrt[m]{y^b} = \sqrt[nm]{x^{ab} \cdot y^{ma}}$$

Of course, it is perfectly evident, but a check against all available copies of algebra texts suggested that it may not have reached the printed page. Nevertheless, Jim, his classmates, and I enjoyed the experience of "proving something that wasn't in the book."

(signed) ROBERT C. O'NEAL  
Quincy Senior High School  
Quincy, Illinois

The January, 1954, issue of THE MATHEMATICS TEACHER has just reached me and it was a delight to note the new format. For years I have felt somewhat depressed by lack of change and it now has such a more attractive appearance that I feel sure many who formerly merely glanced at it, will now be intrigued into more completely perusing it.

(signed) H. C. JACKSON  
Humphrey C. Jackson,  
Retiring President  
Michigan Council of Teachers  
of Mathematics

In my opinion, the listing of references on science, mathematics and religion has no place in THE MATHEMATICS TEACHER. Science and mathematics, *per se*, have nothing to say about religion although individual scientists and mathematicians often do; theology exists for the study of religion. It is unfortunate, in my opinion, when a professional magazine for mathematics teachers publishes material that is so far removed from mathematics.

My compliments on the changes in the magazine.

(signed) JOSEPH R. SLIGO  
741 Melrose Avenue  
Iowa City, Iowa

# Reviews and evaluations

Edited by Cecil B. Read, University of Wichita, Wichita, Kansas

*Basic Mathematics Simplified*, C. Thomas Olivo, Albany, Delmar Publishers, 1953. Cloth, iii + 421 pp., \$3.75.

The purpose of this book is three-fold:

1. The material is organized around everyday affairs.
2. The scope is broad enough to develop the necessary competency for everyday needs.
3. The instructional units include all of the basic principles required in arithmetic, algebra, geometry, and trigonometry.

Each basic-principle unit presents the necessary material for mastering that unit. The illustrations are clear and plentiful. The assignments which follow the step-by-step presentations provide a generous amount of practice to develop the necessary mastery.

Achievement tests are given at the end of each section of basic-principle units. These tests may also be used as pre-tests of the student's ability. Additional study guides and workbooks are available. Workbooks have been provided for electrical, machine, carpentry, sheet metal, automotive, painting, plumbing, printing, masonry, and needle trades.

The book is well organized and the author suggests such practical uses as the following: (1) a textbook in secondary education, (2) a textbook in vocational classes, (3) a textbook for on-the-job training programs, (4) a source book or reference book for teacher training classes and (5) a textbook for adult education programs.

Fourteen pages of useful tables are included in the appendix.—*Winnie Macon, Haskell Institute, Lawrence, Kansas.*

*Core Curriculum Development—Problems and Practices*, Bulletin 1952, No. 5, Grace S. Wright, Washington, United States Government Printing Office, 1952. Paper, iii + 104 pp., \$0.30.

This booklet brings together factual information answering two questions: (1) What is the core program like as it is being developed in the secondary schools in the United States? and (2) How have high schools which have achieved fairly successful core programs attacked the problems which principals report as major concerns? Regardless of the reader's position on the question of the utility of the core curriculum, this study will aid any teacher in objectifying his point of view.—*Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa.*

*Engineering Statistics and Quality Control*, Irving W. Burr, New York, McGraw-Hill Book Co., Inc., 1953. Cloth, vii + 442 pp., \$7.

This book will be of interest primarily to engineers and those in industry. As the title indicates, those topics that are useful in engineering and quality control are included. The major emphasis is on control charts, both for measurements and for attributes. Other statistical methods such as tests of significance, analysis of variance, and correlation techniques are omitted. The author takes great care to make the methods appear reasonable on the basis of empirical evidence for those whose mathematical background is limited, and on the other hand to give mathematical derivations suitable for readers with a year of calculus. It appears to be a well-written book for its purpose.—*Fred W. Lott, Iowa State Teachers College, Cedar Falls, Iowa.*

*A First Course in Functions of a Complex Variable*, Wilfred Kaplan, Cambridge 42, Mass., Addison-Wesley Publishing Company, Inc., 1953. Cloth, vii + 619 pp., \$3.50.

This small book, which contains sufficient material for a one semester undergraduate course, is an exact reprint (except for the correction of some misprints) of chapter 9 of the author's *Advanced Calculus*. As a consequence of its origin, a knowledge of a few special topics such as the theory of line integrals in the plane, and the theory of infinite series is presupposed. The author has made a point of stressing the physical applications of the theory.—*Augusta Schurrer, Iowa State Teachers College, Cedar Falls, Iowa.*

*Foundations of the Nonlinear Theory of Elasticity*, V. V. Novozhilov, Rochester, New York, Graylock Press, 1953. Paper, vi + 233 pp., \$4.

This book, translated from the Russian, is based on a 1947 series of lectures by the author who is a member of the Mathematical-Mechanical Department of Leningrad National University. A nonlinear theory of elasticity, a generalization of the classical (linear) theory, is developed which enables us to treat many new and important problems such as the deformation of elastic bodies which do not obey Hooke's Law, and the large deflection of rods.—*Augusta Schurrer, Iowa State Teachers College, Cedar Falls, Iowa.*

*Functional Mathematics Books 1 and 2*, William A. Gager, Charlotte Carlton, Carl N. Shuster, and Franklin W. Kokomoor, New York, Charles Scribner's Sons, 1953. Cloth, iii+447 pp., \$2.96 each.

These interesting books are the first two of a proposed series of four books for the four years of high school. They are intended to cover the mathematics that all high-school students need for general education and at the same time to lay a good foundation for college mathematics. The word "functional" seems to be used in a double sense: that of usefulness in daily living and that of the development of the concept of functional relationships.

The subject-matter is integrated, with arithmetic, algebra, geometry, and trigonometry running through both books. Because of this integration the material may seem chopped up, with little continuity from chapter to chapter, but such a characteristic is probably unavoidable in this type of course. There are some good examples of stimulating interest through integration, such as introducing a need for square roots in the work on indirect measurement.

The twenty-nine points of the Commission on Post-War Plans of the National Council of Teachers of Mathematics have been used in planning the content. At the end of the second book the twenty-nine points are stated and there are a few pages of practice exercises based on them. Approximate numbers are studied early, and their proper use is kept well in mind throughout the two books. There is a considerable amount of consumer mathematics (taxation, insurance, etc.) in both books. This material is treated well and it is probably true that all students need consumer mathematics and that the mathematics classroom is perhaps the best place to study it. However, many teachers feel that there is very little real mathematics in it, in proportion to the time and effort spent on it, and also there is considerable question whether this subject-matter can be very meaningful to ninth- and tenth-graders. Should they have it later when its use will be more immediate and real to them?

In the method of presentation there is a great deal of experimental discovery encouraged and most conclusions are arrived at by inductive or intuitive reasoning. There is one chapter in Book 2 on "Correct Patterns of Thinking" which is excellent in itself. However, the only deductive reasoning in the two books is a part of that chapter, which seems like very little. With a few exceptions, the concepts and principles are carefully and clearly presented and considerable effort is made to encourage understanding rather than mechanical manipulation. Poor readers might have trouble with the great amount of explanatory material, but in general the explanations are well stated.

Some good special features are the sections called "Keep your tools sharpened" where continuous review of arithmetic is given; the title, "Class activities," on the lists of exercises; the

fairly frequent suggestions for projects and special reports; the review of vocabulary by printing "glossary" words in bold type right in the review questions, not just in a list; and some material on mathematical instruments. The format of the books is excellent.—*Henry Swain, New Trier Township High School, Winnetka, Illinois.*

*How to Lie with Statistics*, Darrell Huff, New York, W. W. Norton & Co., Inc., 1954. Cloth, 6-142 pp., \$2.95.

This delightful little book points out how "the secret language of statistics, so appealing in a fact-minded culture is employed to sensationalize, inflate, confuse and oversimplify." Living as we do in an age where pressure groups of all sorts support their arguments by an appeal to so-called facts, this book is particularly welcome.

The chapter headings will give the spirit of the book. They are: "The Sample with the Built-in Bias," "The Well-Chosen Average," "The Little Figures That Are Not There," "Much Ado about Practically Nothing," "The Gee-Whiz Graph," "The One-Dimensional Picture," "The Semiattached Figure," "Post Hoc Rides Again," "How to Statisticulate," and "How to Talk Back to a Statistic."

Many specific examples of chicanery with statistics are cited. Teachers of high-school mathematics may find it useful as a source of interesting material for their classes. Very little mathematical background is required. In fact, elementary arithmetic and the ability to read simple graphs are sufficient to understand practically all of the book. It is written in a humorous style with numerous cartoons and illustrations. It should be fun for students and teachers alike.—*Fred W. Lott, Iowa State Teachers College, Cedar Falls, Iowa.*

*Intermediate Algebra for College Students* (Rev. ed.), Thurman S. Peterson, New York, Harper & Brothers, 1954. viii+369 pp., \$3.25.

The revised edition of this textbook preserves many of the excellent features of the older edition and adds many new features and problems that should stimulate interest. It gives ample material on factoring and fractions that could be used to advantage with students who have difficulty with these topics. The stated problems are excellent and require the type of analysis so necessary for further mathematical study.

Not least by any means among the excellent features of this book are the review sections. The students have an opportunity in these sections to determine if they have assimilated the material previously studied.

The material in this text is sufficient to overcome the deficiency of a short course in high-school algebra, and also to prepare the student for further study in mathematics.—*Herbert L. Lee, University of Tennessee, Knoxville, Tennessee.*

*An Introduction to the History of Mathematics*, Howard Eves, New York, Rinehart & Co., Inc., 1953. Cloth, xv + 422 pp., \$6.

This book is delightful reading for anyone interested in tracing chronologically the important achievements in the growth and development of a great science from a primitive origin to the rise of the superstructure during the eighteenth and early nineteenth centuries. The book closes with a glimpse of the possibilities of modern mathematics as the logical foundation is deepened and strengthened. With a little extrapolation on the part of the reader he can almost envision the contributions of mathematics to the development of civilization.

The great wealth of knowledge on the history of mathematics makes it necessary for an author of a text on the subject to exercise personal judgment in the selection of material. In Part I of the text the author has chosen wisely from Babylonian, Egyptian, Greek, Hindu, and Arabian contributions. There is an excellent discussion of Greek mathematics which includes some of the problems as exercises. There is a brief concise treatment of Euclid's *Elements* with some mention of their logical shortcomings and the subsequent creation of non-Euclidean geometries.

Part I is concluded with a discussion of the mathematics created by the Hindus and Arabs and also their role in the transmission of learning into Europe. One chapter is devoted to the development of mathematics in Europe from 500 to 1600.

In Part II the rapid growth of mathematics during the seventeenth century is brought into relief. Also it is pointed out that the advance in mathematics parallels other intellectual pursuits—social, economic, and political gains. And finally one sees in this picture the forerunner of the calculus reaching even from the works of Archimedes to its creation by Newton and Leibnitz.

This text is not a biography or a story of the lives of great mathematicians, rather it gives the story of mathematics with personal information as interesting sidelights. One of the most outstanding and valuable features is the inclusion of problems. At the end of each chapter is a set of "Problem Studies." These problems are based on the content of the chapter and furnish the student an opportunity to grasp the importance of, and to develop an appreciation for, the mathematical achievements of past civilizations. With the skilful use of these problems it is possible for the student to get a good course in mathematics as well as about mathematics. This text is especially well adapted to teacher education classes and fills a great need for a text in that area. Anyone interested in training prospective teachers of secondary or elementary mathematics would do well to give this book careful consideration. It should surely be in the professional library of every mathematics teacher.—H. G. Ayre, *Western Illinois State College, Macomb, Illinois*.

*An Introduction to the Theory of Differential Equations*, Walter Leighton, Ph.D., New York, McGraw-Hill Book Co., Inc., 1952. Cloth, viii + 174 pp., \$3.50.

Professor Leighton has made an attempt to get away from the "cook book" type of introductory text which is all too common in this area. The fundamental existence theorem, which is stated early, proved in an appendix, and used throughout the text, is the central idea around which a unified treatment of the material is built. The result is a presentation which makes the subject seem more like a coherent theory rather than a grab bag of tricks. Applications are stressed throughout.—Augusta Schurrer, *Iowa State Teachers College, Cedar Falls, Iowa*.

"Know Your Arithmetic," "Answers to Know Your Arithmetic, with additional suggestions," "Know Your Geometry," "Answers to Know Your Geometry, with additional problems and suggestions," "Know Your Algebra," "Answers to Know Your Algebra," "Know Your Trigonometry," "Wall Chart of Logarithms and Trigonometric Functions," Jacob Hieble, Ithaca, The Thrift Press, 1944. Each paper-covered booklet, 48 pp., \$0.25; Chart \$0.20.

A careful reading of these booklets fails to reveal what population they might serve. They are far too condensed and confused for the high-school student or out-of-school adult. The content is not nearly as clear or well done as comparable "review books" used in secondary schools. They are of no conceivable value to the teacher of mathematics.

Unfortunately, the many vague and incorrect statements in the booklets preclude any recommendation that these booklets be kept in departmental or classroom libraries. "Ratio is the relation of one number to another of the same kind." (p. 9, Arith.) "The area of a trapezoid is  $(a+b)h$ ." (p. 28, Arith.) "Subtracting a negative quantity is the same as adding it." (p. 5, Alg.) "The angle marked out by a straight line is 180 degrees." (p. 2, Geom.) "The midpoints of the sides of any quadrilateral form a parallelogram." (p. 24, Geom.)

It was hoped that the problems would make up for some deficiencies in the text, but this was not so. "If I find \$5, what is my gain in %?" (p. 20, Arith.) "Find  $\sqrt{58,863,869}$ " (p. 35, Arith.) "From a cube of wood, the largest possible sphere is turned, etc." (p. 43, Arith.) It is interesting to comment that the machine by which this turning was done would be much more profitable than the booklets.

A revision of these booklets would also have to include corrections of the method of finding characteristics (all booklets), some indication of how to find the position of the decimal point in slide-rule calculation, and a general clarification of the geometrical constructions in "Know Your Geometry." Too, the trigonometric curves (p. 10, Trig.) have no units on the vertical axis.

The reviewer recommends a rather careful rewriting and correction of these booklets if they are to be recommended.—*Irving Allen Dodes, Chairman, Department of Mathematics, Bronx High School of Science.*

*Math Is Fun*, Joseph Degrazia, New York, Emerson Books, Inc., 1954. Cloth, 5-159 pp., \$2.75.

The first sentence in the Preface probably describes the book. "This book is the result of twenty years of puzzle collecting." This is an excellent book for recreational mathematics. It should be in every school library, for the knowledge required never exceeds high-school mathematics although this does not mean that the problems are all simple. There are 196 problems with answers in the back of the book, and in many cases with hints for solutions. One finds many classical puzzle problems as well as many new ones. One of the most amusing features is the clever set of sketches which accompany the material.

The problems are roughly classified by type. To illustrate, there are chapters entitled: "How old are Mary and Ann?"; "Wolf, goat and cabbage—and other odd coincidences"; "Clock puzzles"; "Trouble resulting from the last will and testament"; "Railroad shunting problems"; "Cryptograms"; "Faded documents" (multiplication and division problems with most of the digits replaced by an arbitrary symbol); "Shopping puzzles"; "Problems of arrangement." One excellent feature of the book is the care with which the author shows how to approach the solution of certain classic problems, thus leading the reader to a method for solving other problems of similar type. As an example of a valuable hint is the suggestion that one can use playing cards as a visual help for solving most railroad puzzles.

On page 47 one wonders if the author means to use the spelling "Tartalea" for the Italian mathematician or whether this was a misprint. Aside from this minor point, no misprints or other errors were noted. Certainly this little book can be highly recommended.—*Cecil B. Read, University of Wichita, Wichita, Kansas.*

*Mathematics for All High School Youth*, (report of basic skills conference-clinics in mathematics), Albany, New York State Education Department, 1953. Paper, 2+108 pp., \$0.50.

The New York State Education Department, as a part of its program for the readjustment of high-school education, sponsored in 1951 a series of conference-clinics at various points in the state to consider ways of improving basic skills in reading and mathematics. These two areas were chosen because of a belief that improvement of basic skills in them could result in the improvement of many related areas. The participants in the mathematics meetings of the conference-clinics included a large number of teachers and administrators of elementary and secondary schools, college teachers, nationally

known leaders in the field of mathematics education, and members of the staff of the State Education Department.

The present report summarizes the discussions of the conference-clinics. The first part describes problems and issues that were considered to be important in the teaching of the basic skills of mathematics that should be useful to nearly all individuals in our society. Various points of view on these problems and issues are discussed. The second part presents suggestions of the chief consultants for initiating readjustments in high-school mathematics. It suggests basic content for differentiated curricula in mathematics and it offers much practical advice on teaching methods. A bibliography is included in an appendix.

This report is a valuable addition to the literature on mathematics in general education at the high-school level. It contains the best short discussion of current issues of secondary education and the place of mathematics in the total program that this reviewer has seen. It should be of interest to all who are concerned with the readjustment of high-school mathematics programs and especially to those who are working to improve basic skills. Many teachers have been hoping for a textbook or a syllabus that will furnish the entire content for a good course in general mathematics. This report will not serve that purpose; indeed, it emphasizes that much problem material should be current and local.—*Gilbert Ulmer, University of Kansas, Lawrence, Kansas.*

*Mathematics in Western Culture*, Morris Kline, New York, Oxford University Press, 1953. Cloth, xxii+484 pp., \$7.50.

This new book is not a textbook; nor is it a history of mathematics in the usual sense, although it certainly has a fine historical approach. According to the author, "The object of this book is to advance the thesis that mathematics has been a major cultural force in Western civilization." This I believe it will do. That is the main reason why I think the book should be read by every teacher of secondary and college mathematics, to say nothing of elementary teachers and others, who have an interest in the place of mathematics in modern civilization.

In the *Foreword* by Professor R. Courant we read:

"After an unbroken tradition of many centuries, mathematics has ceased to be generally considered as an integral part of culture in our era of mass education. The isolation of research scientists, the pitiful scarcity of inspiring teachers, the host of dull and empty commercial textbooks and the general educational trend away from intellectual discipline have contributed to the anti-mathematical fashion in education. It is very much to the credit of the public that a strong interest in mathematics is none the less alive." This book by Professor Kline is one of the first books in English, if not the first, of any im-

portance, to be devoted to the cultural value of mathematics. It demonstrates how one who has the proper background can show in an interesting fashion how mathematics has developed since the time of the early Egyptians and Babylonians.

As the author points out, "Mathematics detached from its rich intellectual setting in the culture of our civilization and reduced to a series of techniques has been grossly distorted." Mathematics has too often been presented as a system of meaningless tasks. The influence of mechanistic psychology, particularly in arithmetic, where "significance, meaning and insight" have been overlooked, has produced a generation of students who do not understand anything about the various mathematical principles they are supposed to employ. Besides, they have very wrong conceptions of the nature and place of mathematics in modern civilization. That is unfortunate. Many laymen today think that anyone interested in mathematics, particularly one who teaches the subject, is to be considered queer in some respects.

I hope that we can get more and more teachers interested in the cultural significance and the universality of mathematics in the coming years. This new book of Professor Kline's, if teachers will read it, should have a great effect upon their understanding of the subject and will enable them to present in a more attractive manner the evolution of mathematical thinking.

It is unfortunate that books of this type are today listed at prices that make it almost impossible for most teachers to buy them. For this reason, it is hoped that such books can be obtained in school and public libraries throughout the country.—*William David Reeve, Professor Emeritus of Mathematics, Teachers College, Columbia University, New York.*

**Editorial Note:** The *New York Times* thought this book of sufficient importance to devote a full page to its review. Those interested may find this in the book review section of the November 15, 1953, issue.—*Cecil B. Read.*

*A Mathematician's Miscellany*, J. E. Littlewood, London, Methuen and Co., Ltd., 1953. Cloth, v+136 pp., 15s.

This offers material covering a wide range—mathematics for the "amateur" as well as the professional; a description of the author's mathematical education; reprints of reviews. The material is too advanced for high schools, but valuable to the university library for many reasons (such as topics for mathematics club programs).—*Cecil B. Read.*

*Plane Trigonometry*, Paul R. Rider, New York, The Macmillan Co., 1949. Cloth, v+180 pp., \$3.

Twelve chapters of the author's *First Year Mathematics for Colleges* constitute the material for this text. The material considers the acute angle before the general angle. To reduce the

functions of any angle to a corresponding function of an angle less than 90 degrees the author uses the unit circle rather than a series of formulas thus emphasizing method rather than memorization of formulas. The chapter on approximate numbers and computation appears quite complete. Vectors are introduced in one section. The tables are four-place except for the table of values of trigonometric functions of an angle given in radians which is a five-place table. The definition for reference angle varies with the value of the angle concerned, hence may be confusing to some students. The misprint occurring on page 41 is continued from *First Year Mathematics for Colleges* and an additional misprint occurs on page 22.

Generally, the discussions are clear and should be easy for the students to understand. This book should prove valuable as a text because of its clarity of discussions and diagrams and its numerous exercises.—*Otho M. Rasmussen, University of Denver, Denver, Colorado.*

*Plane Trigonometry With Four-Place Tables*, 2d Edition, Arthur W. Weeks and H. Gray Funkhouser, New York, D. Van Nostrand Co., Inc., 1953. Cloth, iii+197 pp., \$2.88 with four- or five-place tables; \$2.68 without tables.

This text is a revision of an earlier book by these authors and changes are made in such a way that this edition can be used with the earlier edition. The major changes include a short section on exponential equations at the end of the preliminary section on logarithms, two sets of review exercises spaced through the text, and a final comprehensive review of objective test questions. The text begins with computational material and appears to progress from the easier to the more complex parts of the subject. The acute angles of a right triangle are considered before the general angle is discussed. The last chapter pertains to the applications of trigonometry in navigation, vector quantities, and the resolution of forces. A large number of exercises are provided in all parts of the text. The discussions are such that the student should get an understanding of the subject as limitations on accuracy and use of formulas are discussed.

Negative characteristics are given as  $\bar{1}$ ,  $\bar{2}$ , etc. in all parts of the text and the tables. Separate tables of logarithms are printed for each of the functions. Scales used in the graphs are not consistent and may give a distorted idea of the graphs of some functions.

This text is available with no tables, four-place tables, or five-place tables.—*Otho M. Rasmussen, University of Denver, Denver, Colorado.*

*Problem Book in the Theory of Functions*, Volume I, Problems in the Elementary Theory of Functions; Volume II, Problems in the Advanced Theory of Functions, Konrad Knopp, New York, Dover Publications, Inc., 1948 and 1952. Paper, viii+126 pp. and 138 pp., \$1.25.

These two volumes were designed to furnish problems to accompany the study of "Theory of Functions," Parts I and II, by the same author. They can also be used to accompany comparable texts used in graduate study of elementary and advanced theory of functions.—*Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa.*

*Roger Bacon in Life and Legend*, E. Westacott, New York, Philosophical Library, Inc., 1953. Cloth, vii + 140 pp., \$3.75.

This small book, based largely on a biography written by Emile Charles (Paris, 1861), tends to be technical and give many details including numerous footnotes. The author has made an effort to distinguish between fact and tradition. It is probably as complete a treatment as is available in English and would be valuable for any library. Interesting sidelights on life in the thirteenth century are presented along with material on Bacon. There is relatively little historical material relating directly to mathematics.—*Cecil B. Read.*

*Science Since 1500, a short history of mathematics, physics, chemistry, biology*, H. T. Pledge, New York, Philosophical Library, Inc., 1947. Cloth, 357 pp., \$5.00.

Although the primary emphasis is upon developments since 1500, there is some treatment of the status prior to that time. Emphasis is placed upon the interrelations between the sciences. At various points it is obvious that the author writes for British readers ("Our own Marlowe . . ."). The statement that here an attempt is made for the first time in a general book to discuss nineteenth-century mathematics may be too strong; it depends upon the definition of "general."

Unusual features are charts and maps showing improvement in the accuracy of measurement; the connection of master and pupil; and birthplaces of scientists.

There is probably nothing available which attempts to cover so much in such short space. Certain points to which this reviewer objected (for example, the treatment of Fermat's last theorem on page 47, and the phrase "a Jew convert" on page 187) were found corrected in an errata sheet, the last page in the book. There is no mention of such developments as atomic physics or statistical methods.—*Cecil B. Read.*

*The Teaching of Higher Geometry in Schools*, A Report prepared for the Mathematical Association, London, G. Bell & Sons, Ltd., 1953. Paper, iii + 119 pp.

This is an excellent report. The aim of the authors was to suggest a view of geometry for the consideration of the teacher; to present ideas that will act as a stimulus to the teacher rather than to provide material from which he can make selections for teaching.

The first six chapters should be read by all

teachers of geometry while the entire report should be read by those teaching geometry on the university level.

The report designates a first course and a second course. The syllabus of the first course includes all that can be regarded as suitable for every pupil who takes mathematics as a major subject whether he expects to become a mathematical specialist, a scientist, or an engineer. The report recognizes the fact that all pupils cannot be expected to cover so much ground, hence in Chapters 4-6 an attempt has been made to summarize from actual classroom experiences the content of subject-matter on which it has been possible to draw. The second course discusses the content of subject-matter suitable only for specialists in mathematics and other subjects who have marked mathematical ability. Most of this material is a first year's work in mathematics at a university. The importance of making the transition from geometry taught at the precollege level to that taught at the college level less abrupt is stressed.

Certain trends in the teaching of geometry were noted in the report. Among these trends were: (a) to disregard boundary lines between the various types of geometry; (b) to make analytical geometry of two and three dimensions abstract not spacial; (c) a less formal course in the properties of solids; and (d) an increased interest in topology.—*L. H. Whitcraft, Ball State Teachers College, Muncie, Indiana.*

*A Text Book of Matrices*, Shanti Narayan, Delhi, S. Chand & Company, 1953. Cloth, ii + 289 pp., 7/8/- (price).

This book reflects the urgent need for a textbook for graduate and post-graduate students in mathematics and in science. In addition to a good, standard presentation of the theory of matrices, this book is to be commended for presenting appropriate exercises throughout its pages.—*Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa.*

*Trends in Production of Teaching Guides*, A Survey of Courses of Study Published in 1948 through 1950, Eleanor Merritt and Henry Harap. Division of surveys and field services, George Peabody College for Teachers, Nashville, Tennessee, July 10, 1952. Paper, 31 pp., \$0.50.

This monograph reports current practices and trends in the production of curriculum guides based on research over the period from 1948 to 1950. The study is of a general nature, touching upon the core curriculum, yet it does give a few references to curriculum guides that have been developed in mathematics at the elementary and secondary levels.—*Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa.*

"U.S. Navy Occupational Handbook for Women," "U.S. Navy Occupational Handbook for Men," Bureau of Naval Personnel.

Washington, D.C., 1953. Paper, ii + xiv pp. (women); ii + xv pp. (men).

Although these two publications are Navy manuals, they are primarily for use by school counselors and civil agencies. The complete story of the occupational opportunities and job structures of the U.S. Navy is told in a factual way and could be used as an integral part of the guidance program and as a classroom aid in occupations courses of secondary schools and colleges. The Navy urges young people to get as much schooling as possible, to plan their careers early, and to take subjects that will contribute to the chosen field of work.

The manuals list Vocational Information Briefs of the major job fields, sixty-two for men and twenty-seven for women. Each brief presents a concise statement of what the job is, the duties and responsibilities, qualifications and

preparations, training courses available, and related occupations. A School Subject Index chart cites the counselor or student to the school subjects which contribute most directly to the job. The information would also be of value to students planning civilian occupations.

Qualification for the majority of the job assignments point up the need for training in mathematics. To qualify for some of the jobs it is necessary to make satisfactory scores on: the Navy General Classification Test which measures ability to learn and think; and, the Navy Arithmetic Test which measures ability to use numbers in practical problems. Other job assignments indicate the need for skill in dealing with simple arithmetic problems, particularly those dealing with weights and measures, while others indicate the need for an understanding of algebra, geometry, and trigonometry.—*Olive G. Wear, Fort Wayne Public Schools, Fort Wayne, Indiana.*

## The Second New Jersey Mathematics Institute

*Sponsored jointly by*

THE ASSOCIATION OF MATHEMATICS TEACHERS  
AND RUTGERS UNIVERSITY  
NEW BRUNSWICK, NEW JERSEY  
*July 7-July 16, 1954*

### PROGRAM

Sessions daily except Sunday

Study Groups, 8:30-9:30 A.M.

A-1 Building Mathematical Concepts in the Elementary School (Kindergarten through Grade 3), CHARLOTTE JUNGE and JANE PLENTY.

A-2 Teaching the Reading of Mathematics (Elementary level), ONA KRAFT and an associate leader.

A-3 The Geometry of the Junior High School, MARY ROGERS.

A-4 Algebra As an Indispensable Tool and As a Way of Thinking, WILLIAM BETZ.

A-5 A New Approach to High-School Geometry, VIRGIL MALLORY.

A-6 Construction of Effective Classroom Tests, ALICE GRISWOLD.

General Lecture Series, 9:45-10:45 A.M.

Study Groups, 11:00-12:00 M.

B-1 Building Mathematical Concepts in the Elementary School (Grades 4-6), CHARLOTTE JUNGE and JANE PLENTY.

B-2 (Topic to be announced—of special interest to elementary teachers.)

B-3 The Arithmetic of the 7th and 8th Grades, MARGARET DUNN.

B-4 Teaching the Reading of Mathematics (Junior High Level), ONA KRAFT.

B-5 A Program for the Superior High-School Pupil, RICHARD PIETERS.

B-6 A Second Track for Grades 10-12, GEORGE GARTHWAITE.

Lunch and Open Discussion, 12:00 M.-2:00 P.M.  
Mathematics Laboratories, 2:00-4:00 P.M.

L-1 Laboratory Mathematics—Elementary Level, AMANDA LOUGHREN.

L-2 Junior High School Laboratory, MARGARET DUNN.

L-3 Senior High School Laboratory, AMELIA RICHARDSON.

L-4 Mathematical Instruments and Their Use, CARL SHUSTER.

L-5 The Development of Reasoning Power, JOHN RECKZEH.

Dinner and After-Dinner Addresses, 6:30-8:30 P.M.

The 1954 Institute will be housed at the beautiful New Jersey College for Women Campus in New Brunswick (instead of the Men's Campus as previously announced). A planned social program includes events for each day of the Institute. The Institute carries two points of academic credit, awarded by the Rutgers University School of Education. Costs will be kept moderate: Dormitory (double occupancy), \$12.00; Tuition and registration fees (two points credit), \$27.00 for New Jersey residents, \$33.00 for others; Total Institute fee (no academic credit), \$20.00.

All who are interested in enriching the work being done in mathematics and in finding more effective teaching methods are invited to become members of the Institute. The brochure of the 1954 Institute may be obtained from Director of the Summer Session, Rutgers University, New Brunswick, New Jersey.

## • TIPS FOR BEGINNERS

*Edited by Francis G. Lankford, Jr., University of Virginia, Charlottesville, Virginia.*

*Have you been dissatisfied with your way of handling reviews of previous work?*

*Here are some helpful suggestions.*

### *Handling reviews*

*By Francis G. Lankford, Jr.*

As pupils move from their elementary school work in arithmetic into high-school courses in general mathematics and algebra they do so with eager anticipation of what these new experiences are to be. This is true of those pupils who have not been too successful in arithmetic as well as those who have. The unsuccessful pupil hopes to be able to make a fresh start and the successful one wonders if he will be able to continue his thoroughly satisfying experiences. Both groups are genuinely disappointed if they learn at the outset of their new courses that first they must spend an indefinite period in reviewing old content *before* the new work can begin. Such a review is frequently too rapid for the weak pupil to get much new help, and it is often useless and completely boring for the capable pupil. All of this is equally true of new courses in high school, as pupils move from algebra to geometry and from geometry or algebra to trigonometry.

Many years ago Breslich<sup>1</sup> questioned the value of such reviews in this manner.

Some evidence of the futility of brief formal reviews at the beginning of a course has been secured in classes of the University High School (University of Chicago). In attempting to solve the problem for the first course in the senior high school, the department of mathematics designed a brief course which all ninth-grade pupils had to take during the first weeks of the school year. The instructional materials were obtained by eliminating the signs and the literal numbers from the algebraic exercises in the text-

book that was used in the first-year classes. This material was organized topically, taught, reviewed, and supplemented by practice and drill. In this manner the pupils performed precisely the arithmetical operations which they were to carry out later in the study of algebra.

The results were disappointing. It was found that in the oral and written work in the course the same arithmetical errors and difficulties appeared that had been characteristic of classes of former years. The teachers felt that the solution of the problem had not been found in the method that was used.

The experiment referred to above was continued in the following year, but the solution of the problem was attempted by a different method. It was decided to choose a central project which calls for the use of much arithmetic. As a problem suitable for the purpose, the department selected the study of the properties, areas, and volumes of the common solids. It offers ample opportunities for computational work and gives rise to a large number of arithmetical situations which involve the essential arithmetical processes. The study of the solids supplied a motive which served the students as an incentive to review their arithmetic.

This second suggestion of how to manage the needed review at the outset of a new course is a good one. You may try it with confidence. There are other ways of handling such reviews and of avoiding undesirable by-effects. One such possibility is to teach the arithmetic again through algebraic generalizations. For example much attention may now be given to the associative, commutative and distributive laws governing the operations. Further emphasis may be devoted to the inverse relationships among the operations. These laws and relationships may be expressed algebraically and illustrated with arithmetical numbers.

<sup>1</sup> E. R. Breslich, *Problems in Teaching Secondary School Mathematics* (Chicago: Univ. of Chicago Pr., 1940), pp. 24-25.

### Commutative law

$$A+B=B+A. \quad 17+29=29+17$$

$$AB=BA. \quad 15 \times 20 = 20 \times 15$$

### Associative law

$$(A+B)+C=A+(B+C)$$

$$7+2+11=9+11=7+13$$

$$A(BC)=(AB)C. \quad 8 \cdot 2 \cdot 5 = 8 \cdot 10 = 16 \cdot 5$$

### Distributive law

$$a(b+c)=ab+ac. \quad 6(3+8)=18+48$$

### The inverses

If  $a+b=c$ , then  $c-b=a$  or  $c-a=b$

$$8+2=10, \quad 10-2=8 \text{ or } 10-8=2$$

If  $ab=c$ , then  $c/b=a$  or  $c/a=b$

$$9 \cdot 2 = 18, \quad 18/9 = 2 \text{ or } 18/2 = 9$$

In the case of a division of  $c$  by  $a$  with a quotient of  $b$  and remainder,  $r$ , we may write  $ab+r=c$ .

If  $29/3=9$  with remainder of 2, then  $(3 \times 9) + 2 = 29$ .

These inverses may be used, of course, for checking computations. In providing illustrations and in using them for checking, considerable valuable review will be had and at the same time pupils will begin algebra and experience new content.

A third possibility of handling the review of arithmetic at the outset of high-school courses in general mathematics and algebra is to stimulate pupils to invent new ways of doing arithmetic operations. For example it may be suggested that they learn to add two columns at a time in this manner.

$$\begin{array}{r} 32 \\ 29 \end{array} \quad \begin{array}{r} 32+20=52 \\ 61+40=101 \\ 104+10=114 \end{array} \quad \begin{array}{r} 52+9=61 \\ 101+3=104 \\ 114+4=118 \end{array}$$

If pupils have always used the invert-

and-multiply rule to divide fractions, they may now use this procedure.

$$2/3 \div 3/4 = 8/12 \div 9/12 = 8/9.$$

If they have always used the decomposition method of subtraction, they may now find some satisfaction in considering subtraction additively.

Subtract

517 Think:

294	294 and 6 make 300	6
?	200 more make 500	200
	17 more make 517	17
	Hence $517 - 294 = 223$	223

I am currently working with some teachers in an experimental test of the value of this last suggestion. At first we wondered if pupils would be more confused than helped by several optional approaches. Our preliminary experiences suggest that this is not the case. Many pupils gain much enthusiasm for learning "New ways of doing arithmetic." Moreover, they soon begin to devise ways of their own to try out with their classmates. Not only do we accomplish a review in this manner but frequently fresh enthusiasm for mathematics develops as it now becomes more of an experience in independent thinking.

There is a final suggestion that all needed review should not be completed at the beginning of a new course. As new topics are met they may be accompanied by effective review. This is true of most topics in general mathematics and algebra. Examples are units on measurement, formulas, and insurance in general mathematics; and factoring, fractions, and simultaneous equations in algebra. We may well look for such opportunities optimistically, for surely it is a responsibility of high school mathematics to broaden insights into the operations and concepts of arithmetic. This responsibility will always be present regardless of how well the job is done of teaching arithmetic in the elementary school.

# What's new?

## BOOKS

### JUNIOR HIGH SCHOOL

- Arithmetic for Today 7*, Thomas J. Durrell, Adaline P. Hagaman, and James H. Smith, Columbus, Charles E. Merrill Books, 1954. Cloth, ii+316 pp., \$0.99 net, \$1.32 list.
- Arithmetic for Today 8*, Thomas J. Durrell, Adaline P. Hagaman, and James H. Smith, Columbus, Charles E. Merrill Books, 1954. Cloth, ii+316 pp., \$0.99 net, \$1.32 list.

### HIGH SCHOOL

- Algebra Book 1*, C. A. Smith, W. Fred Totten, and Harl R. Douglass, Evanston, Row, Peterson & Co., 1954. Cloth, iii+500 pp., \$2.72.
- Algebra Notes*, William J. Hazard, New York, Vantage Press, Inc., 1952. Cloth, 1-198 pp., \$2.95.
- Guidance Pamphlet in Mathematics for High School Students* (revision), edited by I. H. Brune, Washington, D.C., National Council of Teachers of Mathematics, 1953. Paper, iii+40 pp., \$0.25.
- Mathematics for Everyday Living*, Adele Leonhardy and Vivian B. Ely, New York, D. Van Nostrand Co., Inc., 1954. Cloth, iii+470 pp., \$2.96.

### COLLEGE

- Calculus*, George B. Thomas, Jr., Cambridge, Addison-Wesley, 1953. Cloth, 1-614 pp., \$6.50.
- Differential Equations in Engineering Problems*, Mario G. Salvadori and Ralph J. Schwarz, New York, Prentice-Hall, Inc., 1954. Cloth, v+432 pp., \$8.65.
- Elementary Differential Equations*, Lyman M. Kells, New York, McGraw-Hill Book Co., Inc., 1954. Cloth, v+266 pp., \$4.
- Elements of Statistics*, H. C. Fryer, New York, John Wiley & Sons, Inc., 1954. Cloth, v+262 pp., \$4.75.
- A First Course in Ordinary Differential Equations*, Rudolph E. Langer, New York, John Wiley & Sons, Inc., 1954. Cloth, v+249 pp., \$4.50.
- Introduction to College Mathematics* (2d ed.), Carroll V. Newsom and Howard Eves, New York, Prentice-Hall, Inc., 1954. Cloth, iii+408 pp., \$5.75.
- Rinehart Mathematical Tables, Formulas and Curves* (enlarged ed.), compiled by Harold D. Larsen, New York, Rinehart and Company, Inc., 1953. Cloth, viii+280 pp., \$2.

*Theory of Equations*, Cyrus Colton MacDuffee, New York, John Wiley & Sons, Inc., 1954. Cloth, v+120 pp., \$3.75.

### MISCELLANEOUS

- Handbook of Probability and Statistics With Tables*, Richard Stevens Burington and Donald Curtis May, Sandusky: Handbook Publishers, 1953. Cloth, v+332 pp., \$4.50.
- A History of the Theories of Aether and Electricity*, Sir Edmund Whittaker, New York, Philosophical Library, Inc., 1954. Cloth, v+319 pp., \$8.75.
- Instructional Leadership*, Gordon N. Mackenzie and Stephen M. Corey, New York, Bureau of Publications (Teachers College, Columbia University), 1954. Cloth, v+209 pp., \$3.25.
- Math Is Fun*, Joseph Degrazia, New York, Emerson Books, Inc., 1954. 159 pp., \$2.75.
- Mathematics, Minus and Plus*, Leeds K. Field, New York, Pageant Press, 1953. Cloth, 15 pp., \$2.

## MODELS

- W. M. Welch Scientific Co.,  
1515 Sedgwick Street,  
Chicago 10, Illinois
- |  |              |
|--|--------------|
| Conic Sections, ellipse, parabola, and hyperbola | \$14.50 each |
| Hyperbolic Sections                              | 15.00 each   |
| Cone with Circular Section                       | 7.00 each    |
| Dandelin's Cone                                  | 21.00 each   |
| Dissectible Pyramid                              | 10.00 each   |
| Trisectible Prism                                | 25.50 each   |
| Sectioned Four-sided Prism                       | 10.50 each   |
| Sectioned Six-sided Prism                        | 11.00 each   |
| Sectioned Cylinder                               | 9.50 each    |
| Tetrahedron                                      | 6.00 each    |
| Hexahedron                                       | 6.50 each    |
| Octahedron                                       | 7.00 each    |
| Dodecahedron                                     | 17.50 each   |
| Icosahedron                                      | 17.00 each   |
| Coaxial Cylinders                                | 16.00 each   |
| Cylinder through Cylinder                        | 13.00 each   |
| Cone through Cone                                | 12.00 each   |
| Sphere with Great Circle Sections                | 7.50 each    |
| Sphere with Spherical Sector                     | 14.00 each   |
| Segmented Sphere                                 | 15.00 each   |
| Hemisphere with Spherical Pyramid                | 20.00 each   |
| Decimeter Cube                                   | 13.00 each   |
| Binomial Cube                                    | 17.50 each   |
| Pythagorean Theorem Model                        | 20.00 each   |

Continued on page 367

## • WHAT IS GOING ON IN YOUR SCHOOL?

*Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and  
Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana.*

*Here is presented an account of how one teacher provides "that little extra"  
for her seventh grade. Also presented are some ideas  
from a classroom teacher on high-school mathematics clubs.*

### *Desert for seventh-graders*

*Contributed by Julia E. Diggins, Paul Jr. High School, Washington, D.C.*

After a nourishing meal from the seventh-grade course of study, we have desert. The promise of desert makes the meal more palatable and without completing the meal of course, the desert might be withheld. The ideas for desert come from a little book by H. V. Baravalle.<sup>1</sup>

After studying lines, angles, simple geometric forms, and the protractor, we use that skill to do the cut-outs described with pictures in the book. We go further by discovering original designs, hundreds of them. The originals are made by varying the number of folds, and the angles at which the bars are drawn to form the designs.

Each original design must have its recipe (formula to the mathematician), and the children have the fun of directing the class according to their original formulas to cut out some particularly interesting design that has been created.

These designs are of such variety and beauty that when posted on colored paper they make attractive room decorations.

The first showing to select the best work is done by dipping the cut-out in a pitcher

of water and sticking it on the blackboard. Enthusiasm and rivalry fill the air. The formulas for complicated patterns are in demand. Some wag will say, "What are you doing, cutting out doilies in math class?"

The answer is to point out the perfect squares, triangles, hexagons, stellar hexagons, octagons, and dodecagons, and show the neat little formulas written on the designs by seventh-graders.

The desert after studying the circle, bisecting lines and arcs, dropping and erecting perpendiculars is first to divide a circle into twelve equal parts. Using this drawing as a pattern, place under it several sheets of paper. Stick a pin point through the papers and you will have four or five papers with a circle divided into twelve equal parts to work with.

Number the 12 points. Connect the numbered points in the following manner:

1 to 3, 2 to 4, etc. and you will get 2 hexagons.

1 to 4, 2 to 5, etc. will give 3 squares.

1 to 5, 2 to 6, etc. will give 4 triangles.

1 to 6, 2 to 7, etc. will give a continuous stellar polygon.

Many valuable discoveries described in *Geometry* occur in these drawings.

\*Then divide the circle into 24 equal

<sup>1</sup> H. V. Baravalle, *Geometry* (Garden City, L.I.: Adelphi College, 1900).

parts. Use this as a pattern and proceed to make more by pin-sticking.

With the papers prepared with the 24 divisions of the circle here are some recipes (formulas):

Call radius—"r."

Call the original circle "O. C."

Use the points on O. C. as centers of the circles to be drawn.

1. Draw circles from each of the 24 points on O. C. with  $r$  equal to the radius of O. C.

2. Draw circles from each of the 24 points on O. C. with radius equal to  $\frac{1}{2} r$  of O. C.

3. Draw circles from each of the 24 points on O. C. with radius equal to  $2 r$  of O. C.

4. Place a point within O. C. and draw circles from each of the 24 points on O. C. with the radius extending from the points on O. C. to the point within the circle.

5. Do the same with a point placed on the circle.

6. Do the same with a point outside the circle.

The children will work hours on this kind of homework. They will not tolerate sloppy geometric drawing. Their work will be precise and neat. Let them color these drawings according to their own taste, and many original and surprising patterns will be the result.

There are many other fine ideas in the little book *Geometry* particularly on the spiral.

There is good sound geometry, inspiration for original ideas, and the opportunity for personal discovery and appreciation of mathematical truths on the part of the children. I have had no experience with any group regardless of I.Q. not lapping up this desert, and going on to the next meal with a better appetite.

## The Archimedean

*Contributed by Sister Anne Agnes, C.S.J., Rosati-Kain High School, St. Louis, Missouri*

For the past three years, Rosati-Kain has had a very active Mathematics Club under the name of "The Archimedean." It is composed of girls who take advanced mathematics as well as those who cannot fit it into their program but who are happy to have this means of maintaining interest in the subject.

A committee of three is appointed to take care of the program for each meeting. A variety of activities has been presented. These have taken the form of skits, radio broadcasts, explanation of some advanced theory, biographical sketches of mathematicians, history of mathematics, quiz programs, number games, riddles, and puzzles.

The following constitution has been drawn up according to the rules of parliamentary law. Certain articles which

have been quite effective in the functioning of the club are included here.

### CONSTITUTION

#### ARTICLE I. Name

*Section 1.* This Mathematics Club shall be called the Archimedean of Rosati-Kain High School.

#### ARTICLE II. Members

*Section 1.* Membership in this Club is limited to such students of Rosati-Kain from the sophomore to the senior year who have an average of at least 85 per cent in mathematics and who show a marked interest in the subject. High standards of conduct must be maintained.

*Section 2.* The November meeting shall be open to prospective members at which time the Constitution will be read and the purpose of the Club explained. Suggestions for projects shall be made whereby future members can demonstrate their interest.

*Section 3.* Procedure in Admission. Above projects are given to some members of the Club on a

fixed date. Any member has the right of nominating any number of students to the Club on the condition that the nomination be signed by two other members. These nominations together with the projects shall be handed to the Committee on Admissions who shall carefully examine the qualifications and vote on the nominees. Those duly elected shall be notified by the Committee and initiated at the December meeting.

#### ARTICLE III. Officers

*Section 1.* The officers of this Club shall be a President, a Vice-President, a Secretary-Treasurer, and a Librarian, which officers shall be elected from the ten members having the greatest number of honor points.

*Section 2.* Elections shall be held at the first meeting of the school year.

*Section 3.* Each officer shall serve until his successor is duly elected. Vacancies may be filled temporarily by presidential appointment until the next regular meeting of the Club.

#### ARTICLE IV. Meetings

*Section 1.* Regular meetings shall be held once a month on a day to be agreed upon by the members.

#### ARTICLE V. Honor Points

*Section 1.* An honor pin shall be earned by students who have accumulated 100 points.

*Section 2.* Attendance at a meeting earns 4 points.

*Section 3.* Faithful discharge of the duties of an officer earns 10 points or less depending upon manner of discharge of same.

*Section 4.* Serving on a committee earns 3 points.

*Section 5.* Performance at a meeting earns 5 points.

*Section 6.* Besides above, President has power to grant honor points when judged proper.

*Section 7.* An unexcused absence from a regular meeting forfeits 4 points.

*Section 8.* An excused absence forfeits 2 points.

*Section 9.* If member is at home ill on day of regular meeting, no points are forfeited.

*Section 10.* Failure to perform some assigned task forfeits 5 points.

*Section 11.* Member is dismissed if number of honor points becomes negative. Warning is served on the zero level.

### What's new?

*Continued from page 364*

#### FREE MATERIALS

##### *Statistical Methods*

Monroe Calculating Machine Company,  
Orange, New Jersey  
Booklet.

##### *Quality Control*

Monroe Calculating Machine Company,  
Orange, New Jersey  
Booklet.

##### *Circles of Light*

Photo News Service, General Electric,  
Schenectady, New York  
Posters, 14"×17"; black and white.

##### *Atom Tester*

Photo News Service, General Electric,  
Schenectady, New York  
Posters, 14"×17"; black and white.

##### *Money Management. Your Equipment Dollar*

Household Finance Corp., 919 North Michigan Avenue, Chicago 11, Illinois  
Booklet, 36 pp.

##### *The New York Stock Exchange*

New York Stock Exchange, Broad and Wall Streets, New York, N. Y.  
Booklet, 45 pp.

##### *Time Flies*

Air World Education, 80 East 42nd Street, New York, N. Y.  
Booklet, 12 pp.

##### *Steinmetz: Latter Day Vulcan*

Public Relations; General Electric Company, 1 River Road, Schenectady 5, New York  
Booklet, 12 pp.

##### *Invitation to Youth (Careers in Life Insurance)*

Institute of Life Insurance, 488 Madison Ave., New York 22, N. Y.  
Single copies free to educators.

*What is going on in your school?* 367

## • AFFILIATED GROUPS

*Edited by Mary C. Rogers, Chairman Affiliated Groups,  
Roosevelt Junior High School, Westfield, New Jersey.*

*One of the never-ending functions of groups of mathematics teachers is to teach  
and talk about the general education values of their subject. Affiliated groups  
occupy key positions for the propagation of such discussions.*

### Goals in algebra<sup>1</sup>

*Contributed by Jackson B. Adkins, Committee on Affiliated Groups,  
The Phillips Exeter Academy, Exeter, New Hampshire*

Algebra is, of course, essential for the continued study of mathematics. To study algebra as a terminal course in mathematics is quite another matter. We here discuss the goals of such a terminal course and assume that the course will be given in the ninth grade. The student of mathematics will observe that there is no real conflict between the goals herein presented and the goals for pupils who intend to continue the study of mathematics. There might be a difference in intensity but not in the general pattern of the course.

This pattern is formed around three major goals. The ninth-grade algebra course should achieve:

1. Greatly increased skill in and knowledge and understanding of arithmetic.
2. Some appreciation of the postulational basis of mathematics.
3. Understanding of and some skill in using algebra as a device for answering questions about quantitative relationships.

All the desirable "practical" aspects of arithmetic are embodied in the first goal. The second and third goals are admissible only if one is willing to admit that the impact of mathematics on our culture is so

great and so pervasive that no person can claim a liberal education consonant with our times if he is wholly ignorant of the field of mathematics. A beautiful and convincing exposition of this thesis is presented by Morris Kline in his recent book, "Mathematics in Western Culture."

The immediate result of introducing letters to stand for numbers is to force the attention of the pupil onto the postulates that govern the combining of numbers. Thus the routine response to " $12 \times 7$ " gives way, in the algebraic equivalent, " $(a+b)c$ ," to an examination of the distributive postulate in mathematics and a much fuller understanding of the statement that " $12 \times 7$ " is the same as " $(10+2)7$ ." This more sophisticated approach to arithmetic opens up the subject and illuminates it in a way not usually even "sensed" in the seventh and eighth grades. This is true, not because the teachers of those grades didn't talk this way, but because the pupil wasn't "ready" for it. He was too young. The great advantage of algebra, rather than just another arithmetic course, as a vehicle for this strengthening of the arithmetic is that an entirely fresh setting is provided. This excites the pupil and produces a magnificent "clinching" of all elementary arithmetic that the old setting could not possibly achieve.

<sup>1</sup> Reprinted by permission of the Bulletin of The National Association of Secondary School Principals, from the May, 1954, issue.

The extraction of square root, the factoring of large numbers with the resultant review of the multiplication tables, the review and drill in fractions, all achieve a different coloring and an added stimulus when they appear as necessary aspects of the solution of equations. Percentage is simplified, clarified, and made useful when taught in the language of algebra.

The associative postulate for multiplication,  $a(bc) = (ab)c$ , must be "brought up to the surface of the mind" and explored when the equation  $2/3(3x - 5/2) = 7$  is to be multiplied by 6. The axioms of addition, subtraction, multiplication, division, and substitution throw further light on the postulational basis of mathematics and at the same time provide a thorough review of the fundamental operations of arithmetic.

The algebra teacher is impelled to point out, from time to time, the fundamentals of a logical system: undefined terms, defined terms, postulates, the "if-then" pattern of thought. Thus, the laws of operation with signed numbers are never proved. Their plausibility and usefulness may be illustrated in many ways. But in the final analysis they are seen to be postulates. The "if-then" pattern of thought governs the solution of equations. The symbol, " $3x$ ," is *defined* to mean "3 times  $x$ ." The symbol, " $x^3$ ," is *defined* to mean " $x \cdot x \cdot x$ ." At this level we usually accept "greater than" as an undefined term. We don't worry much about defining "equality."

The third goal, to see algebra as a device for answering questions about quantitative relationships, ties the subject plainly to mathematics as a whole and opens up to the curious fourteen-year-old great vistas of unexplored knowledge. We elaborate the goal in the following manner.

1. Problems (in mathematics) deal with quantities.

2. If the problem can be solved, these quantities are related.

3. These relations, when translated into algebraic symbols, produce equations.

With these ideas in the background the pattern of thinking becomes clear: We must find out what quantities a problem is talking about. We then put down symbols for the quantities. Then state the relation (or relations) between the quantities—using the symbols. Equations result. The crux of the matter is, of course, stating the relation between the quantities.

Relations between quantities always appear in a problem in one or both of two ways. Either they are explicitly stated by the words of the problem or they are implied by the words of the problem. In the first case the job is a straight translation job from the marks in the book which we call words to the marks (letters, equals signs, etc.), employed for convenience in algebra. If the relationship is implied then the words of the problem must be a sufficient cue (like a cue in a play) to cause us to remember the relationship. Take a simple example.

The length of a rectangle is twice its width. The perimeter is 100 inches. Find the dimensions.

If we denote the length by  $L$  and the width by  $W$  then the first relationship between these quantities is explicitly given by the words "length is twice the width" and straight translation produces the equation  $L = 2W$ .

The second relationship is implied by the words "rectangle" and "perimeter." These words must be a sufficient cue to cause us to remember "perimeter of a rectangle is the sum of twice its length and twice its width." The translation of this remembered relationship produces  $2L + 2W = 100$ . With these two equations the student of algebra can then go on and perform the necessary substitutions, additions, subtractions, etc. to reach the answers.

This pattern of thinking is completely general. Thus, formulas take their place in the pattern as convenient devices for

remembering relationships. The great bulk of the study of elementary geometry falls into place as merely a study of the relationships between the various parts of figures. Trigonometry likewise is exhibited, in its elementary aspects, as a further study of relationships between parts of a triangle. It becomes plain to the student that a problem is "difficult" because the relationships between the quantities are obscure.

The whole study of mathematics takes on form and sense if the pupil gets hold of this general pattern of thought in elementary algebra. He has a framework to which more and more ideas and items of information will cling. Every subsequent course in mathematics will broaden and strengthen this fundamental idea and will clarify the concept of postulational thinking. If he studies no more mathematics, here is a permanent and understandable answer to the question "What is mathematics all about?"

No subject in the curriculum suffers more than algebra from the obscuring of the woods by the trees. It is so easy, under inept handling, to get bogged down in the details of the subject that the pupil often fails to see where he is headed. The "woods" that must constantly be kept in front of the pupil's eyes are:

1. Algebra is a device for dealing with quantitative relationships.
2. Mathematics has a postulational basis.

It is fatal to let these drift into the background of the child's thought. When they do, the subject becomes pointless, uninteresting, and a doubtful aspect of general education. When they stay in the foreground so that each day's work "sticks" somewhere on this framework, then there is a steady access of understanding and a broadening appreciation of the role of mathematics in our culture.

The nature of the subject-matter de-

mands steady and hard work from its students. At the level under discussion it does not demand more than average intelligence. It demands concentration, neatness, and habits of checking each step before proceeding. The earnest school administrator who recognizes the pervasive nature of mathematics in modern culture can get results from good teachers if he permits them to demand work from their pupils. The softness of much present day schooling that requires little or no "homework" and no results that take steady effort is the greatest enemy of the subject.

Algebra puts arithmetic into a challengingly different setting. It distinguishes between "truth" that is relative to postulates and the "absolute truth" of the child's religious beliefs. It enables even the poorest student who has no intention of pursuing mathematics to understand in broad outline what the great body of civilization's mathematical knowledge is all about. It generates that satisfaction and emotional stability that derive from knowledge that a hard job has been well done. In its social context algebra takes its place as the sine qua non for introducing each generation to the vast body of mathematical information that the race must pass on.

As the accumulated knowledge of the race expands it becomes increasingly important—and difficult—to select those items that should be passed on to all people. Only the kind of algebra we are talking about here can be justified for all people. When the trees obscure the forest and the pedant pursues recondite elements of the subject because they amuse him, then the real goals of the subject are lost and its place in general education becomes untenable. But, if properly taught by good teachers who have a setting in which they can function, it opens the path, in exciting fashion, to one of the great achievements of the human mind.

## *Which way mathematics?*

*by H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa*

The problems set forth in the four feature articles in this issue of *THE MATHEMATICS TEACHER* should be of interest to all mathematics teachers. The fact that the enrollment of the secondary schools is expected to increase 2.8 million during the next ten years not only raises problems of housing and teacher supply but it raises curricular problems which are not easy to answer. These problems are not only problems for the secondary-school teacher; they are rapidly becoming the problems of the college teacher. Increased enrollments are bound to decrease the degree of selectivity in the college population as well as in the secondary-school population.

That the high schools, and colleges, will find it necessary to further provide differentiated courses for students of different interests and abilities is quite obvious if they are to accommodate all who knock at their doors. That simultaneously there will be some condensation of materials for the student of the upper ability group is a natural expectation. Yet there is a danger in the process of condensation. If there is simply a condensation of present materials—such as teaching plane and solid geometry in one year's time—more harm may result than good. This group needs above all else to be introduced to ideas which have more mathematical power than the ideas which generally occur in some parts of the present program. Neither will a reshuffling of topics within the present program suffice. The remedy must lie in more drastic measures than either of the above. As Kenneth O. May

points out, the quality of the program must receive as close a scrutiny as the time factor. Condensation and reshuffling alone will not produce the desired result.

There is yet another danger. In the hurry to get things done a teacher may forget that to teach both more and better mathematics he must know more about how adolescents learn. The urge to teach more mathematics may cause the teacher to defeat himself. Is it not true that learning under pressure encourages rote learning? To what extent does rote learning block the learning of real mathematics? To really learn mathematics the student must have some time to "mull over" mathematical ideas; to discuss these ideas with his teacher and with other students, to examine the ideas much as one would examine a new and interesting toy, and, above all, to use the idea in a number of different situations.

The fact that the learning process must be more carefully considered is clearly indicated by the mistaken notion that some teachers have as to the part that drill plays in the learning of mathematics. To a great degree, and in a real sense, learning is completed before drill starts. This is especially true if the drill is of the type provided in many textbooks. There is an incubation period when ideas are grasped. During this period—a period of real educational import—drill cannot make a contribution. Drill, of the usual type, can only increase the student's confidence that he can handle the idea after he has it. Of course, drill will also decrease his reac-

tion time when he is confronted with a situation which requires the use of the idea.

One more comment in this brief space. The shortcomings of the elementary school and high-school programs are but a reflection of the shortcomings of the college programs. Certainly there can be no reorientation of the high-school program in mathematics if the change in the high-

school program is not accompanied, or preceded, by a similar soul searching on the college level. This merely points up the fact that no group is to blame for the present situation if it is bad. The reorientation of mathematics is a common problem for all teachers of mathematics whether they are in the elementary school, secondary school, undergraduate college, or even, perhaps, graduate schools.

## Instructions to authors of manuscripts

Manuscripts that come to the editorial office of THE MATHEMATICS TEACHER should be prepared according to the following directions. Much labor will be saved by both author and editor if the considerations, listed below, are kept in mind when preparing manuscripts for publication in THE MATHEMATICS TEACHER.

1. All manuscripts and editorial correspondence should be sent to:

H. Van Engen, Editor  
THE MATHEMATICS TEACHER  
Iowa State Teachers College  
Cedar Falls, Iowa

2. All manuscripts should be typewritten, double or triple spaced, on  $8\frac{1}{2}'' \times 11''$  good quality paper. The manuscripts should be typed so as to leave one-inch margins on both sides (one and one-half inches is even better).

3. References and footnotes should be numbered consecutively and placed at the bottom of the page. Prepare footnote references as follows:

To a book:

<sup>1</sup> Francis G. Lankford, Jr. and John R. Clark, *Basic Ideas of Mathematics* (New York: World Book Company, 1953), p. 115.

To an article in a periodical:

<sup>2</sup> Jack D. Wilson, "Arithmetic for Majors?" THE MATHEMATICS TEACHER, Vol. XLVI (December 1953) p. 260.

To a yearbook:

<sup>3</sup> Maurice L. Hartung, "Motivation for Education in Mathematics," *Twenty-first Yearbook*, The National Council of Teachers of Mathematics, Washington, D.C., 1953, chap. II, pp. 42-69.

To a technical bulletin, pamphlet, or similar publication:

<sup>4</sup> *Guidance Pamphlet in Mathematics*, The National Council of Teachers of

Mathematics, Washington, D.C., 1953, p. 16.

4. If a bibliography is included at the end of the article, use the following guide: For a book:

CLARK, CHARLES E. *An Introduction to Statistics*. New York: John Wiley & Sons, Inc., 1953.

For an article in a periodical:

WILSON, JACK D. "Arithmetic for Majors?" THE MATHEMATICS TEACHER, Vol. XLVI (December 1953), pp. 560-64.

5. All drawings should be on good quality white paper in black india ink. Also use india ink for letters and numbers which appear in the drawing. Make drawings somewhat larger than the final print in the magazine. When mailing drawings insert cardboard in envelope to prevent bending.

6. The order for reprints must be placed when the galley proofs are returned to the editorial office. A blank for ordering reprints is furnished with each set of galley proofs. Authors of feature articles will receive four copies of the magazine from the editorial office.

7. Changes made in galley proofs that are sent to the author for proofreading are expensive. Indicate only corrections to make the galley proofs agree with your manuscript.

8. Return galley proofs promptly.

# NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

## Annual Summer Meeting

*The National Council of Teachers of Mathematics, in  
Co-operation with the National Education Association*

Teachers College, Columbia University, New York City

June 28, 1954

### THEME

## *Man's Right to Knowledge and the Free Use Thereof*

9:00-9:40 A.M. Registration—Teachers College Main Hall. All those attending are urged to register. There is no fee.

9:40-9:45 A.M. Welcome: HOWARD F. FEHR, Teachers College, Columbia University, New York

9:45-11:30 A.M. *Mathematics as Knowledge*

Presiding: ELIZABETH SIBLEY, President, Association of Mathematics Teachers of New York City

"Mathematics as a Cultural Heritage," H. Von Baravalle, Adelphi College, New York

"Mathematics as an Applied Tool," C. C. Hurd, International Business Machine Company, New York

12:00-1:15 P.M. Luncheon—Teachers College Cafeteria. No reservations are necessary. Meals are at choice of individual.

1:30-2:30 P.M. Panel Discussion: *The Use of Mathematics in Problem-Solving*

Presiding: MARGARET DUNN, President, Association of Mathematics Teachers of New Jersey

Moderator: JOHN KINSELLA, New York University

Elementary School: GERTRUDE HILDREDTH, Brooklyn College, New York

Junior High School: FLORENCE POTTER, New York State Department of Education, Albany, New York

Senior High School: LESTER W. SCHLUMPF, Andrew Jackson High School, New York

2:45-4:00 P.M. *The Teacher's Need for and Use of Mathematical Knowledge*

Presiding: MYRON F. ROSSKOPF, Teachers College, Columbia University, New York

In Elementary Teaching: BEN SUELZT, Cortland State Teachers College, Cortland, New York

In Secondary School Teaching: DAVID R. DAVID, Montclair State Teachers College, New Jersey

Chairman: HOWARD F. FEHR

Co-Chairmen: MYRON F. ROSSKOPF, ELIZABETH SIBLEY, MARGARET DUNN

Hospitality: Arranged through the New York City and the New Jersey State Associations of mathematics teachers.

# Fourteenth Summer Meeting

*The National Council of Teachers of Mathematics*

University of Washington, Seattle, Washington

August 22, 23, 24, 25, 1954

CONVENTION THEME

## *Mathematics in Focus*

*Sunday, August 22, 1954*

3:00–8:00 P.M. Registration—Men's Residence Hall, 1101 Campus Parkway

*Monday, August 23, 1954*

8:00 A.M.–4:30 P.M. Registration—First Floor, Physics Hall

8:00 A.M.–4:30 P.M. Exhibits—School and Commercial—Second Floor, Physics Hall

8:45–10:00 A.M. General Session—Guggenheim 224

Welcome: HENRY SCHMITZ, President of the University, University of Washington, Seattle, Washington

*Mathematics in Focus*, F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

### SECTIONAL MEETINGS

10:15–11:45 A.M. **Elementary**—Physics 224

*Some Contributions of Recent Research to the Teaching of Arithmetic*, DAN DAWSON, Stanford University, Stanford, California

*How an Understanding of Our Number System Makes Certain Arithmetic Processes Meaningful*, CLIFFORD BELL, University of California, Los Angeles, California

10:15–11:45 A.M. **Secondary**—Johnson 56

Panel Discussion: *Recent Curriculum Developments in Secondary Mathematics*

Chairman: W. VIRGIL SMITH, Public Schools, Seattle, Washington

Discussants: DALE CARPENTER, Los Angeles City Schools, Los Angeles, California; WILLIAM H. GLENN, Pasadena City College, Pasadena, California; KENNETH BROWN, U.S. Office of Education, Washington, D.C.; CATHERINE A. V. LYONS, The University School, Pittsburgh, Pennsylvania

10:15–11:45 A.M. **Applications**—Bagley 211

Panel Discussion: *Mathematics and Related Curriculum Areas*

Chairman: MILTON GOLD, Office of State Superintendent of Public Instruction, Olympia, Washington

Discussants: ATHOL R. BAILY, University of Washington, Seattle, Washington; E. R. WILCOX, University of Washington, Seattle, Washington; JAMES MOUNT, Garfield High School, Seattle, Washington; GRANT I. BUTTERBAUGH, University of Washington, Seattle, Washington

10:15–11:45 A.M. **Program for the Gifted Child**—Physics 148

Panel Discussion: *Report and Discussion of the Ford Foundation Program*

Chairman: LESTA HOEL, Public Schools, Portland, Oregon

Discussants: LAURA SCOTT, Jefferson High School, Portland, Oregon; CAROLINE T. PAIGE, Cleveland High

School, Portland, Oregon; GARFORD GORDON, San Francisco, California; VIRGINIA BARTELL, Grant High School, Portland, Oregon; LLOYD WILLIAMS, Reed College, Portland, Oregon; WILLIAM MATSON, Lincoln High School, Portland, Oregon

**10:00-11:45 A.M. College and Teacher Education—Physics 214**

*Topology, a Generalized Geometry*, PAUL WHITE, University of Southern California, Los Angeles, California

*A Desirable Master's Degree Program for Prospective Teachers of Secondary Mathematics*, JOHN L. MARKS, San Jose State College, San Jose, California

*Monday Afternoon*

**SECTIONAL MEETINGS**

**1:30-2:30 P.M. Elementary—Physics 314**

*Effective Use of Multi-Sensory Aids and Other Resource Materials*, EDITH DAVIDSON and ZELLA STEWART, Public Schools, Seattle, Washington

**1:30-2:30 P.M. Junior High School—Physics 334**

*Effective Use of Multi-Sensory Aids and Other Resource Materials*, ALICE H. HAYDEN, University of Washington, Seattle, Washington

Demonstrators: HARRIET DOHENY, Franklin High School, Seattle, Washington; CALVIN SCHMID, University of Washington, Seattle, Washington; W. CHESTER ULLIN, Kitsap County Schools, Bremerton, Washington

**1:30-2:30 P.M. Algebra—Physics 148**

*Some Needed Changes in High School Algebra*, HENRY VAN ENGEL, Iowa State Teachers College, Cedar Falls, Iowa

*Problem-Solving in Algebra*, LEOTA C. HAYWARD, Colorado A and M College, Fort Collins, Colorado

**1:30-2:30 P.M. Geometry—Physics 250**

*Creating Theorems in Geometry*, MAUDE HOLDEN, Central City, Nebraska  
*Postulating Geometry Theorems*, JOHN

BULLER, Manhattan High School, Manhattan, Kansas

**1:30-2:30 P.M. Junior College—Physics 252**

Panel Discussion: *A Well-balanced Program for Junior College Mathematics*

Chairman: D. GRANT MORRISON, Office of State Superintendent of Public Instruction, Olympia, Washington

Discussants: ALFRED F. SECKEL, Olympic Junior College, Bremerton, Washington; VINCENT COATES, Centralia Junior College, Centralia, Washington

**1:30-2:30 P.M. College and Teacher Education—Physics 246**

*Advanced Treatment of Elementary Topics for Secondary Teachers*, H. M. GELDER, Western Washington College of Education, Bellingham, Washington

*Commutative and Associative Laws*, A. R. JERBERT, University of Washington, Seattle, Washington

**GROUP MEETINGS: Theme—"How I Teach It"**

**2:45-4:15 P.M. Elementary—Physics 254**

*How I Teach Children Arithmetical Understandings and Skills*, DOROTHEA JACKSON, Public Schools, Seattle, Washington

*How I Teach Estimating and Mental Computation*, IRENE SAUBLE, Public Schools, Detroit, Michigan

*How I Teach Addition and Subtraction*, DORA BROSIUS, Lakeview Schools, Lakeview, Oregon

**2:45-4:15 P.M. Junior High School—Physics 148**

*How I Teach Percentage and Its Applications*, ELIDIA SALVERSON, Sharples Junior High School, Seattle, Washington

*How I Teach Mathematical Concepts by Means of Curve Stitching*, EVA A. CRANGLE, Bryant Junior High School, Salt Lake City, Utah

*How I Teach the Concept of Angular Measurement with Multi-Sensory*

- Aids*, F. F. SCHEPMAN, North Bend, Oregon
- 2:45-4:15 P.M. **Senior High School**—Johnson 56
- How I Teach Pupils to Study Mathematics*, LYLE MARY WHEELER, Astoria High School, Astoria, Oregon
- How I Teach Computation with Approximate Data*, HARRY CHARLES-WORTH, East High School, Denver, Colorado
- How I Teach Understanding of Numbers and Equations*, WILLIAM A. TUCKER, John Marshall Junior High School, Seattle, Washington
- 2:45-4:15 P.M. **Algebra**—Physics 250
- How I Teach Signed Numbers*, NOEL WALTHERS, Denny Junior High School, Seattle, Washington
- How I Teach Graphs of Linear Equations*, HERBERT H. LEWIS, Queen Anne High School, Seattle, Washington
- How I Teach Solving Equations by the "Upstairs-Downstairs" Method*, STEPHEN J. CHRISTOPHER, Clallam Bay, Washington
- 2:45-4:15 P.M. **Geometry**—Physics 214
- How I Teach Geometry with Handmade Models*, HARRIET DOHENY, Franklin High School, Seattle, Washington
- How I Teach Overlapping Triangles*, W. GENEVIEVE GORRELL, Queen Anne High School, Seattle, Washington
- How I Teach Geometry with the Use of Multi-Sensory Aids*, JOHN SCHACHT, Bexley High School, Bexley, Ohio
- 2:45-4:15 P.M. **Teacher Education**—Physics 260
- How to Prepare Teachers to Teach General Mathematics*
- From the viewpoint of the classroom teacher, ROY ENGLUND, Centralia, Washington
  - From the viewpoint of the mathematics education instructor, ANNA S. HENRIQUES, University of Utah, Salt Lake City, Utah;
- H. FREE JAMISON, San Jose State College, San Jose, California
- 2:45-4:15 P.M. **College**—Physics 258
- On the Improvement of College Teaching*, C. B. READ, University of Wichita, Wichita, Kansas
- Some Discrepancies in Mathematical Concepts and Definitions*, CHESTER G. JAEGER, Pomona College, Claremont, California
- 2:45-4:15 P.M. Films—Administration 401
- 5:00 P.M. Salmon Bake—Martha Washington School for Girls, 6612 57th Avenue South, Seattle, Washington
- Under supervision of ERNA GUNTHER, Executive Officer, Department of Anthropology, University of Washington, Seattle, Washington
- Program: Roger Ernesti Indian Dancers with commentary by ERNA GUNTHER
- Tuesday, August 24, 1954*
- 8:00 A.M.-5:00 P.M. Registration—First Floor, Physics Hall
- 8:00 A.M.-5:00 P.M. Exhibits—School and Commercial—Second Floor, Physics Hall
- 8:30-9:45 A.M. General Session—Guggenheim 224
- How Big Is Infinity?* IVAN NIVIN, University of Oregon, Eugene, Oregon
- SECTIONAL MEETINGS
- 10:00-11:45 A.M. **Elementary**—Physics 254
- Suggestions on the Enrichment of Teaching Arithmetic*, MARGUERITE BRYDEGAARD, San Diego State College, San Diego, California
- Some Impacts of Psychological Theories on Arithmetic*, PETER L. SPENCER, Claremont College, Claremont, California
- The Genius of the Decimal System of Computation and Notation*, LOUIS E. ULRICH, William T. Sherman School, Milwaukee, Wisconsin
- 10:00-11:45 A.M. **Guidance**—Bagley 211

(Sponsored by Illinois Council of Teachers of Mathematics)

Panel Discussion: *Opportunities and Responsibilities for Guidance in Mathematics*

Chairman: ROBERT T. ALEXANDER, Senior High School, Macomb, Illinois

Discussants: *Guidance from the Standpoint of the Administrator*, HILDEGARDE M. ROMBERG, Oliver H. Perry School, Chicago, Illinois; *Working with the School Counselors*, JESSIE ORRELL, Lincoln High School, Seattle, Washington; *Guidance in the Classroom*, VIRGINIA TERHUNE, Proviso Township High School, Maywood, Illinois; *Making Use of the Guidance Pamphlet, and Free and Inexpensive Materials*, NELLIE ALEXANDER, Edwardsville High School, Edwardsville, Illinois; *What the Student Should Know about Mathematical Needs and Job Opportunities*, C. N. FUQUA, Champaign High School, Champaign, Illinois

10:00-11:45 A.M. **Curriculum**—Johnson 56

Panel Discussion: *An Integrated Four-Year Program in Mathematics vs. the Traditional Sequence (a) Organization and Content of Grades Nine and Ten*

Chairman: SAMUEL A. FRANCIS, San Mateo Junior College, San Mateo, California

Discussants: SIGURD WENAAS, Public Schools, San Francisco, California; BLANCHE SMITH, Lewis and Clark High School, Spokane, Washington; WILLIAM GLENN, Pasadena City College, Pasadena, California; MARIE S. WILCOX, Thomas Carr Howe High School, Indianapolis, Indiana

10:00-11:45 A.M. **Senior Competency**—Physics 148

(Sponsored by Southern Oregon Council of Teachers of Mathematics)

Panel Discussion: *A Mathematics Competency Course for Seniors*

Chairman: ISABELLE BRIXNER, Klamath County Schools, Klamath Falls, Oregon

math County Schools, Klamath Falls, Oregon

Moderator: EVA BURKHALTER, Klamath Union High School, Klamath Falls, Oregon

Discussants: *What Background Can We Expect of Our Students?* DORA BROSIUS, Lakeview Schools, Lakeview, Oregon; *What Criteria Shall We Use for Judging Competence?* EVA BURKHALTER, Klamath Union High School, Klamath Falls, Oregon; *What Does the Community Expect of High-School Seniors?* ROBERT OLIVER, Roseburg High School, Roseburg, Oregon; *Effective Classroom Procedures*, STANLEY KENDALL, Henley High School, Henley, Oregon; *Use of Instructional Materials*, BEULAH ELLIOTT, Altamont Junior High School, Klamath Falls, Oregon

10:00-11:45 A.M. **College and Teacher Education**—Physics 246

*Topics from Modern Algebra for the Prospective High-School Teacher*, HAROLD M. BACON, Stanford University, Stanford, California

*Past and Present Trends in Teaching the Calculus*, SISTER MARY FELICE, Mount Mary College, Milwaukee, Wisconsin

## Tuesday Afternoon

### SECTIONAL MEETINGS

1:30-2:45 P.M. **Arithmetic**—Physics 254

*Arithmetic Clinic*, F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

1:30-2:45 P.M. **Algebra**—Physics 250

*Using Group Dynamics in Algebra*, HUBERT B. RISINGER, Rutgers University, New Brunswick, New Jersey  
*General Educational Values in Algebra*, RUTH LANA, Eagle Rock High School, Los Angeles, California

1:30-2:45 P.M. **General Mathematics**—Physics 148

*A Re-examination of General Mathematics*, F. G. LANKFORD, JR., Uni-

versity of Virginia, Charlottesville, Virginia

*Meeting the Needs of the Slow Learner*, AGNES HEBERT, Clifton Park Junior High School, Baltimore, Maryland

1:30-2:45 P.M. **Junior College**—Physics 252

Panel Discussion: *Issues in Junior College Mathematics*

Chairman: KENNETH THIESSEN, Skagit Valley Junior College, Mt. Vernon, Washington

Discussants: *Current Curriculum Problems*, CLELA D. HAMMOND, El Camino Junior College, El Camino College, California; *Mathematics for the Specialist*, ARTHUR J. HALL, San Francisco State College, San Francisco, California; *In-service Training for Junior College Teachers*, L. CLARK LAY, John Muir Junior College, Pasadena, California

1:30-2:45 P.M. **College and Teacher Education**—Physics 246

*Advanced Topics from Geometry for High-School Teachers*, DAVID DEKKER, University of Washington, Seattle, Washington

*Trends in the Teaching of Applied Mathematics*, T. S. PETERSON, Portland Extension Center, Portland, Oregon

EXCHANGE OF IDEAS GROUPS: Theme—"How Do You Teach It?"

(The primary purpose of these groups is to give the members of the groups an opportunity to raise questions and to discuss issues—no "set" speeches.)

3:00-4:15 P.M. **Elementary**—Physics 254

Leader: W. I. LAYTON, Stephen F. Austin State College, Nacogdoches, Texas

"How Do You Teach It?" Topics to be suggested by the group.

3:00-4:15 P.M. **Junior High School**—Physics 148

Leader: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey

"How Do You Teach It?" Topics to be suggested by the group.

3:00-4:15 P.M. **Algebra**—Physics 258

Leader: OSCAR SCHAAF, Ohio State University, Columbus, Ohio

"How Do You Teach It?" Topics to be suggested by the group.

3:00-4:15 P.M. **Geometry**—Physics 250

Leader: SYLVIA VOPNI, Public Schools and University of Washington, Seattle, Washington

"How Do You Teach It?" Topics to be suggested by the group.

3:00-4:15 P.M. **College and Teacher Education**—Physics 246

*The History of Mathematics as a Teaching Aid*, PHILLIP JONES, University of Michigan, Ann Arbor, Michigan

*College Mathematics in the Seventh Grade*, H. CHATLAND, Montana State University, Missoula, Montana

3:00-5:00 P.M. Films—Administration Building 401

6:30-9:00 P.M. Banquet—Men's Residence Hall

Greetings: SAMUEL E. FLEMING, Superintendent of Schools, Seattle, Washington

*Where Are the Educational Wastelands*, FRANCIS F. POWERS, Dean of School of Education, University of Washington, Seattle, Washington

Wednesday, August 25, 1954

8:30-12:00 M. Registration—First Floor, Physics Hall

8:30-12:00 M. Exhibits—School and Commercial—Second Floor, Physics Hall

8:30-9:45 A.M. General Session—Guggenheim 224

*The Impact of Industry on Mathematics*, R. E. GASKELL, Boeing Airplane Company, Seattle, Washington

GENERAL SECTIONAL MEETINGS

10:00-11:45 A.M. **Supervision**—Physics 250

Panel Discussion: *Improving the Teaching of Mathematics Through Supervision*

Chairman: MARY CLANFIELD, Public Schools, Longview, Washington

Discussants: MARY CLANFIELD, Public Schools, Longview, Washington; LESTA HOEL, Public Schools, Portland, Oregon; CECIL HANNAN, St. Helens School, Longview, Washington; CATHERINE A. V. LYONS, The University School, Pittsburgh, Pennsylvania; ERNEST EDGERTON, Cleveland High School, Seattle, Washington; IRENE SAUBLE, Public Schools, Detroit, Michigan

10:00-11:45 A.M. **Affiliated Groups—Physics 254**

Chairman: ELIZABETH ROUDEBUSH, Public Schools, Seattle, Washington

*Understandings of Arithmetical Principles Needed by the Elementary Teacher*, MAURICE KINGSTON, University of Washington, Seattle, Washington

*Directing Learning in Arithmetic*, C. RICHARD PURDY, San Jose State College, San Jose, California

*Achievement Levels in Grades Seven and Eight*, SIDNEY AINSWORTH, Wisconsin High School, Madison, Wisconsin

10:00-11:45 A.M. **Curriculum—Johnson 56**

Panel Discussion: *An Integrated Four-Year Program in Mathematics vs. the Traditional Sequence (b) Organization and Content for Grades Eleven and Twelve*

Chairman: S. L. MERRIAM, Garfield High School, Seattle, Washington

Discussants: PHILLIP P. STUCKY, Roosevelt High School, Seattle, Washington; ROBERT SEAMONS, Yakima Valley Junior College, Yakima, Washington; S. E. BOSELLY, JR., Franklin High School, Seattle, Washington; FRED KRAMLICH, Lewis and Clark High School, Spokane, Washington

10:00-11:45 A.M. **Mathematical Literacy—Physics 148**

*Developing Mathematical Literacy in America's Youth*, MILTON W. BECKMAN, University of Nebraska, Lincoln, Nebraska

*Generalizations in Mathematics and in General Education*, CLARENCE H. HEINKE, Capital University, Columbus, Ohio

10:00-11:45 A.M. **College and Teacher Education—Physics 246**

*The Emphasis on Postulates in Teaching Algebra*, ROSS A. BEAUMONT, University of Washington, Seattle, Washington

*The Role of the Supervising Teacher in Mathematics Education*, HOMER BOROUGHS, JR., University of Washington, Seattle, Washington

*Teaching the Prospective Teachers of Elementary Mathematics*, ALMA JEAN KILGOUR, Provincial Normal School, Vancouver, B.C.

12:30 P.M. Luncheon—Men's Residence Hall

Presiding: ELIZABETH ROUDEBUSH, Public Schools, Seattle, Washington

Speaker: CARL B. ALLENDOERFER, Executive Officer, Department of Mathematics, University of Washington, Seattle, Washington

Program Chairman: H. GLENN AYRE, Western Illinois State College, Macomb, Illinois

Co-chairmen on Local Arrangements: ELIZABETH ROUDEBUSH, Seattle, Washington and SYLVIA VOPNI, University of Washington and Public Schools, Seattle, Washington

The program chairman would like to take this opportunity to express sincere thanks to all those who have contributed to the creation of this program. The list of names is too long to print here, but recognition should go to all the officers of the National Council with special mention of the fine contribution made by SYLVIA VOPNI, ELIZABETH ROUDEBUSH and CARL B. ALLENDOERFER. Among the others due special thanks are LESTA HOEL, EVA BURKHALTER, MARY CLANFIELD, MARY C. ROGERS and the affiliated groups, particularly those of Washington, from Oregon, and Illinois; and all others participating in the program.

## ANNOUNCEMENTS

### *Registration*

Everyone attending meetings or accompanying a member is expected to register. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, and members of the press are not charged a registration fee, but should register.

*Registration Fee* for members of NCTM, MAA, or elementary teachers is \$1.50. The fee for nonmembers is \$2.50.

*Advanced Registration* should be made as early as possible. Please use the accompanying Advanced Registration and Reservation Form and mail it (with your check made payable to the University of Washington) to the Office of Short Courses and Conferences, 318 Administration Building, University of Washington, Seattle 5, Washington, before August 10, 1954. Upon arrival, those who have registered in advance should check in at the Registration Desk.

*The Registration Desk* will be in the Men's Residence Hall, 1101 Campus Parkway on Sunday, August 22; during the meeting there will be a registration desk located in Physics Hall, University of Washington campus.

### *Housing*

*Advanced Registrations*—Accommodations will be available for men, women, and family units in the Men's Residence Hall, a new twelve-story building, located at 1101 Campus Parkway near Roosevelt Way at the north end of University Bridge. Reservations may be made in advance for August 22, 23, and 24.

*Without Advanced Reservations*—A limited number of reservations for housing and special events will be available at the time of the meeting. Accommodations for August 25 may be made on an individual basis with the Men's Residence Hall during the meeting.

### *Food Service*

*Meals* will be served in the Men's Residence Hall, for both residents and non-residents, beginning with breakfast on Monday, August 23, and ending with the Luncheon on Wednesday, August 25.

*The Salmon Bake and Entertainment* Monday evening will be at Martha Washington School for Girls on the shores of Lake Washington. Courtesy transportation for this event will be arranged by local committees.

No dinner will be served at the Men's Residence Hall Sunday or Wednesday evenings.

### *Costs*

*Meals and Room*—A package charge of \$17.00 per person covers meals, beginning with breakfast Monday morning through Wednesday noon, and room for the three nights of August 22, 23, and 24. This includes the Salmon Bake, Banquet, and Luncheon.

*The room charge* is based on double occupancy of rooms with twin beds. The management of the dormitory prefers to have rooms occupied in this manner. Those who reserve a room for single occupancy will be charged \$3.00 per night instead of \$2.00, making the total for meals and room \$20.00.

*The package charge* is determined by the time of registration and departure. Persons not arriving on Sunday or indicating at time of registration that they will not remain for the entire meeting will be charged for only such services as are included between the time of their arrival and time of their departure. It will not be possible to refund the price of such items as a single omitted meal.

*Individual meal tickets* may be purchased at the following prices: Breakfast, 77¢; Luncheons \$1.15; Wednesday Luncheon, \$1.39; Salmon Bake and Entertainment, \$2.50; Banquet, Tuesday evening, \$2.50.

## Recreation

*Scenic Beauties of Pacific Northwest*—Advanced information and descriptive materials on what to see and do will be sent to those requesting it on the Advanced Registration and Reservation Forms.

*Tours*—Hosts will guide visitors arriving at Men's Residence Hall on Sunday on tours of Seattle and area. Monday evening's outdoor party will include a scenic drive.

An information committee will have available suggested entertainment for families during the meeting—such as ferry trips, water excursions, etc.

*Golf, Museums, and Libraries* will be available. (Small fee for golf.)

*Clothing*—Visitors to the Pacific Northwest, even in August, should pack a jacket or wrap for water trips or evenings out of doors.

*No formal occasions* will be scheduled in connection with the summer meeting.

## Exhibits

*Schools' Exhibits* of teaching aids, mathematical models, instruments, classroom materials, and student projects are invited. Teachers planning to exhibit materials should communicate in advance with Mr. Richard Bennett, Garfield High School, Seattle 22, Washington, before June 1, 1954, to arrange for space. Exhibitors will be responsible for transportation, assembling and dismantling exhibits. Materials may be left in advance with the Department of Mathematics, Physics Hall, University of Washington, for storage until the time of the meeting.

*Commercial Exhibits* of textbooks and commercial teaching aids will be located in Physics Hall. Commercial exhibitors should make arrangements with Mr. M. H. Ahrendt, Executive Secretary, The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C., not later than July 15, 1954.

*Films and Filmstrips* of interest to

teachers of mathematics will be shown as indicated in the program at the Film Center, 401 Administration Building, University of Washington campus. Requests or suggestions regarding films should be addressed to Mr. William R. Clyde, Roosevelt High School, Seattle 5, Washington, by June 1, 1954.

*Request*—Speakers and other participants in the program who need projection equipment or other materials should communicate not later than August 10, 1954, with the Office of Short Courses and Conferences, 318 Administration Building, University of Washington, Seattle 5, Washington.

## Transportation

*Air, Rail, Bus, and Steamship* all serve Seattle. Those wishing to enjoy scenic beauties of the Pacific Northwest by auto will probably approach Seattle via Highway 10 or 99.

Guests arriving on Sunday, who indicate on the Advanced Registration and Reservation Form that they wish to be met, should inform the Office of Short Courses and Conferences, University of Washington, Seattle 5, Washington, of the means of transportation and expected time and place of arrival before August 10, 1954. Air passengers, upon their request, will be met at the Olympic Hotel.

Seattle Transit Lines 7 and 8, which pass the Men's Residence Hall, serve both railway and bus depots.

## Communications

*Mail and Telegrams* for registrants should be addressed in care of The National Council of Teachers of Mathematics, Men's Residence Hall, 1101 Campus Parkway, Seattle 5, Washington. The telephone number of the Men's Residence Hall is Melrose 1677.

*Certificates of Attendance*—Those who wish to certify attendance at these meetings may obtain a certificate at the Registration Desk on Wednesday, August 25, at the close of the meeting.

## ADVANCED REGISTRATION AND RESERVATION FORMS

Fill out the following form completely and mail to Office of Short Courses and Conferences, 318 Administration Building, University of Washington, Seattle 5, Washington, before August 10, 1954. Make checks payable to the *University of Washington*.

Mr., Miss, Mrs. \_\_\_\_\_  
Last name
First name
Initial

Mailing Address \_\_\_\_\_ City \_\_\_\_\_ State \_\_\_\_\_

Member NCTM \_\_\_\_\_ MAA \_\_\_\_\_ Student \_\_\_\_\_ Exhibitor \_\_\_\_\_

Interest: Elementary \_\_\_\_\_ Secondary \_\_\_\_\_ Teacher Training \_\_\_\_\_ Other \_\_\_\_\_

Institution: \_\_\_\_\_

*Housing:* The following dormitory reservations are for the three nights of August 22, 23, 24. Accommodations for the night of August 25 can be made individually with the Men's Residence Hall during the meeting. Accommodations are available for family units and for individuals.

\_\_\_\_\_ single room occupancy, \$3.00 per night for three nights. .... \$ \_\_\_\_\_  
 (number)

\_\_\_\_\_ twin-bed room, \$2.00 per person per night for three nights. ....

I am alone but prefer to room with \_\_\_\_\_

Name and relationship of others in party: (Give ages of children.)

Arrival date: August \_\_\_\_\_, 1954 \_\_\_\_\_ A.M./P.M. I expect to arrive in Seattle by boat \_\_\_\_\_,  
 bus \_\_\_\_\_, auto \_\_\_\_\_, plane \_\_\_\_\_, train \_\_\_\_\_.

Departure date: August \_\_\_\_\_, 1954.

*Meals:* Members of families and friends of those attending the meeting are welcome to all social and recreational events.

\_\_\_\_\_ complete set(s) of meals at \$11.00. (This includes *all* meals from breakfast, August 23, through luncheon, August 25.) .... \$ \_\_\_\_\_

Additional individual meals may be purchased at the Men's Residence Hall during the meeting, with the exception of the following special meals for which *Reservations Must Be Made in Advance:*

\_\_\_\_\_ Salmon Bake, Monday Evening, August 23, at \$2.50. .... \$ \_\_\_\_\_

\_\_\_\_\_ Banquet, Tuesday, August 24, at \$2.50. ....

\_\_\_\_\_ Closing Luncheon, Wednesday, August 25, at \$1.39. ....

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### Vector Representation

*Continued from page 322*

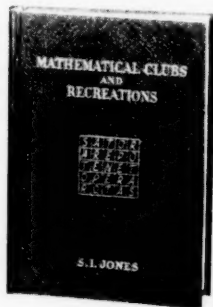
lengths of  $OA$  and  $OB$ , taking the magnitude of the vector  $OD$  of the divisor  $4+3i$  as a unit, represent the real and imaginary components of the quotient. Thus it will be observed that  $OD$  is contained three

times on  $OA$  and two times on  $OB$ , or the quotient is  $3-2i$ . The negative sign of the imaginary component is evidenced by the fact that it is measured along the negative direction of the axis of imaginaries. Care should be taken in determining the proper signs of the components of the quotient.

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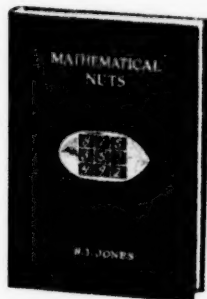
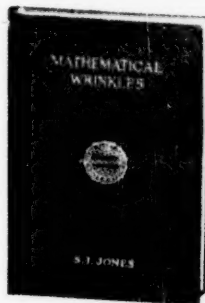
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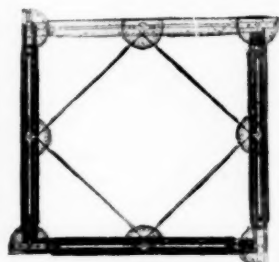
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